



The **new South-African Volatility Index**
new SAVI

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In 2007, the SAVI was launched as an index designed to measure the market's expectation of the 3-month market volatility. The SAVI soon became the benchmark for measuring the market sentiment, and in this light can be thought of as a market "fear" index.

Two years later, in 2009 the Johannesburg Stock Exchange updated the SAVI to reflect a new way of measuring the expected volatility, one that is consistent with the theoretical framework, risk-management and the way traders trade options. The new SAVI is calculated as the at-the-money volatility adjusted for the volatility skew as determined by the actively traded options in the market.

The aim of this note is to introduce the new SAVI calculation method, and briefly discuss the benefits of the new SAVI.

¹ From Financial Chaos Theory: surf to www.quantonline.co.za

² From the JSE and Safex



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1. SAVI

The SAVI was launched, in 2007, as an index designed to measure the market's expectation of the 3-month volatility. The SAVI is based on the FTSE/JSE Top40 index level and it is determined using the at-the-money volatilities. Since it is well documented that there exist a negative correlation between the underlying index level and its volatility, the SAVI can be thought of as a “fear” gauge³. See Fig. 1.

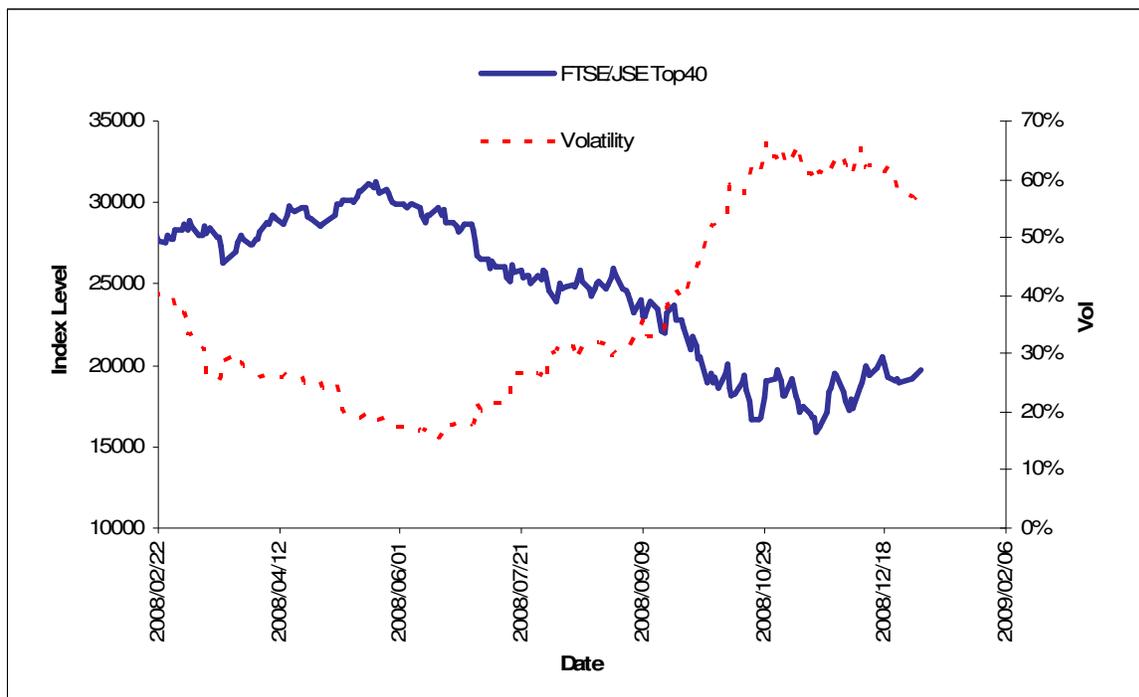


Figure 1: FTSE/JSE Top40 index level and its volatility. When the Alsi40 falls the volatility rises. In this setting, volatility is seen as a “fear” gauge.

Currently, the SAVI is calculated on a daily basis, via polling⁴ the market. The polled at-the-money volatilities are then used to calculate the 3-month at-the-money volatility. The average 3-month at-the-money volatility as determined from the polled volatilities, are then published as the SAVI. For more information on the SAVI see the references [1] and [2].

³ Fear gauge in the sense that high volatility are usually associated with a bear market.

⁴ Polling involves contacting the market participants and obtaining their at-the-money volatilities.

2. The New SAVI

The SAVI is updated two years later, in 2009, to reflect a new way of measuring the expected 3-month volatility. The new SAVI is also based on the FTSE/JSE Top40 Index, but it is not only determined using the at-the-money volatilities but also using the volatility skew. Given that the volatility skew is the market's expectation of a crash, the new SAVI can be thought of as a more efficient "fear" gauge, since it incorporates a market crash protection volatility premium⁵.

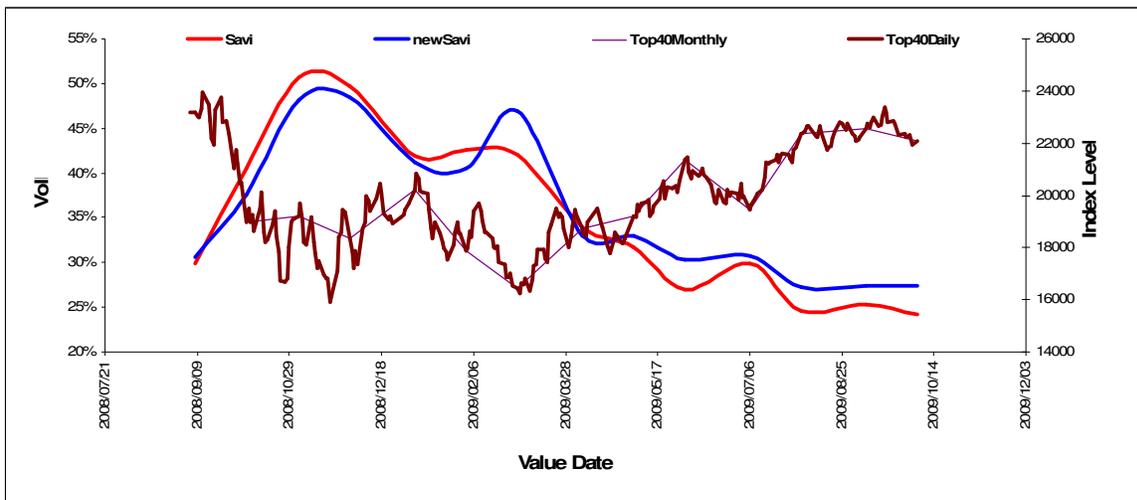


Figure 2: The SAVI and the new SAVI. The FTSE/JSE Top40 index level is also plotted. The new SAVI are slightly different to the SAVI due to the contribution of the skew with the new SAVI.

3. Calculating the new SAVI

The new SAVI are not a polled volatility measurement⁶. The new SAVI is calculated as the weighted average prices of calls and puts⁷ over a wide range of strike prices, that expires in 3-months time. In short,

$$newSAVI = \sqrt{\sum_{i=1}^{n=F} w_{iP} P_i(K_i) + \sum_{i=n}^{\infty} w_{iC} C_i(K_i)} \quad (1)$$

Here F is the current (on value-date) forward of the FTSE/JSE Top40 index level, determined using the risk-free interest rate and dividend yield. F marks the price boundary between the liquid put options $P_i(K_i)$, and call options $C_i(K_i)$ with strikes K_i . The prices of

⁵ This crash protection premium is sometimes referred to as the volatility skew convexity premium.

⁶ This minimises the chances that the calculated volatility index value can be manipulated, by the polled volatility contributors.

⁷ Using calls and puts to find the price of volatility is allowed given that option prices (especially at-the-money options) are directly proportional to their input volatility.

the calls and put options are determined using the traded market volatility skew that expires in 3 month's time.

The 3 month (T) volatility skew, $\sigma_K(0, T)$, is determined using the time weighted interpolation function (with N_1 and N_2 being the days to the near skew, and next nearest skew, from the 3 month skew expiry date, respectively) defined by

$$\sigma_K(0, T) = \sqrt{\left\{ T_2 \sigma_K^2(0, T_2) \left[\frac{N_1}{N_2 - N_1} \right] + T_1 \sigma_K^2(0, T_1) \left[\frac{N_2}{N_2 - N_1} \right] \right\} \frac{N_0}{N_3}}$$

Here, N_0 is the number of days in the year (365 is the South African convention), and N_3 is the number of days from the value date to the 3 month date.

The weights used in equation (1) are that published by Derman et al [3]. The Derman⁸ weightings are piecewise linear recurring option weightings;

$$w_{iP} = \frac{f(K_{i+1}) - f(K_i)}{K_i - K_{i+1}} - \sum_{j=0}^{i-1} w_{jP}, \text{ and } w_{iC} = \frac{f(K_{i+1}) - f(K_i)}{K_{i+1} - K_i} - \sum_{j=0}^{i-1} w_{jC}$$

Where the log-contract is defined by

$$f(F_T) = \frac{2}{T} \left[\frac{F_T}{F_0} - \log \frac{F_T}{F_0} - 1 \right]$$

The new SAVI evaluation methodology for implied volatility measurement using the thinly traded Top40 futures option data, has been tested extensively. It was found that with a strike spacing, $K_i - K_{i+1}$, of 10 index level points leads to negligible approximation errors within the strike range of 70% and 130% option moneyness. Safex therefore calculates the new SAVI using a strike spacing of 10 index level points, and a strike range of 70% - 130% option moneyness.

4. Benefits of the new SAVI

We now describe some stylize facts arising from determining the new SAVI using expression (1);

- The closer the option strikes are to the at-the-money strike the higher the weight assign to the particular option, i.e. the near-the-money volatilities are regarded as more important than the far out-the-money volatilities. Note that if the volatility skew is flat expression (1) reduces to using only the at-the-money volatilities only which is in line with the old SAVI measurement⁹.

⁸ Derman et. al [1], derived these weights in order to fairly price a volatility swap.

⁹ This is consistent with the SAVI calculation, except for (usually small) differences that arises from the fact that a polled volatility measurement are not always the same as a traded volatility measurement.

- The further the option strikes are away from the at-the-money strike, the lower the weights assigned to that particular option. This means that the far out-the-money volatilities are still incorporated into the calculation but their influences are minimal. Remember, the far out-the-money volatilities define the skew¹⁰, and therefore expression (1) includes all possible information from the volatility skew in the determination of the 3 month volatility.

As a result of the new SAVI calculation method the following benefits are present

- The new SAVI calculation includes information of the volatility skew which is in line with the fact that volatilities do not only depend on the time dimension, but it also depends on the strike level dimension. The new SAVI therefore fully incorporates all the dimensions of volatility.
- The new SAVI calculation method is a weighted average of traded option prices, and thereby abandons using the Black-Scholes implied volatility directly. The result of the modification is a model-free volatility index.

The new SAVI, as measured using expression (1), is therefore not only a measure of the 3 month at-the-money volatility, but it is more precisely a measure of the 3 month at-the-money volatility *adjusted for the contribution from the volatility skew*. Calculating the new SAVI in this way ensures that information of the volatility skew (valuable especially to option and other volatility traders) is included in the calculation of the volatility index.

5. The new SAVI as an Asset Class

The new SAVI calculation is based on the market implied volatility skew, and is therefore a more systematic approach to calculating the 3 month implied volatility. A logical question is whether the new SAVI can be utilised as an asset class. The answer to this question is a definite “yes”.

An exposure to the market volatility can be obtained by investing/trading in a variance future. The variance future is a standardised contract that obligates the holder to buy or sell variance (volatility squared), at a predetermined variance strike level. Obtaining exposure to the new SAVI via a variance future does not only provide a pure exposure to the market volatility, but it also allows a volatility investor to determine the price of volatility consistent with risk-neutral evaluations¹¹. Variance futures is a topic of another technical note.

¹⁰ In fact, with the Als40 market the traded options are very sparse in at-the-money options, and hence the deep out/in the money traded options mainly do define the Als40 skew.

¹¹ This is precisely because working with variance (the squared of volatility) makes risk-neutral evaluation of the volatility derivative possible, since variance is additive. Variance future is a topic of another technical note.

6. References

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