\[ I_{\text{comp},t} = \sum_{i=1}^{5} w_i I_{i,t} \]

Money Market Index
May 2008
Introduction

The market expressed a need for an improved index against which money market portfolios could be benchmarked as the accrual methodology applied in available indices made it difficult for asset managers to match the index. The design and composition of the indices covering a range of short term interest rate maturities, is the result of the collaboration between the Bond Exchange of South Africa Limited (BESA) and the Investment Management Association of South Africa (IMASA). Prior to the design, the needs of the investors were fully researched.
Objectives of the Index

BESA and IMASA jointly prepared a questionnaire and distributed it to a group of fund managers for completion. The purpose of this questionnaire was to present a set of options from which the market can make a selection such that a broad consensus can be reached for the construction of a new money market index. The Exchange consolidated the responses received from the fund managers and designed the Money Market Index (MMI) with the objective to provide a benchmark for the money market investors in South Africa that replicates or mimics market behavior.

To achieve this objective, the index was designed to provide:

- A 9 Month JIBOR rate; in addition to the existing 1 Day, 1 Month, 6 Months and 12 Months rates.
- Duration of 90 days in line with money market requirements.
- Daily frequency of trading.
- Date convention of modified following.
- JIBOR rate as input data.
- Cash flows discounted off the BESA Money Market (BMM) Curve.

Index Construction Methodology

The MMI will be constructed from five rates, an overnight, 3 month, 6 month, 9 month and 12 month JIBOR rates. Paper is bought at the JIBOR rate on business days at the above mentioned maturities and held to maturity. As such, five sub-indices are maintained within the index. Each instrument in the given index is marked-to-market daily off the BMM Curve. The sub-indices will be weighted according to market capitalisation based on long term market supply. The average return is then calculated over sequential days and linked to the previous day’s index value to generate the current day’s index value. A composite index, calculated as the weighted average of these sub-indices, will also be generated.

Index Characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Date</td>
<td>03 January 2006</td>
</tr>
<tr>
<td>Base Index</td>
<td>100</td>
</tr>
<tr>
<td>Pricing</td>
<td>Each instrument in the given index is marked-to-market daily off the BMM Curve.</td>
</tr>
<tr>
<td>Publishing Frequency</td>
<td>The index will be published daily after 11 am.</td>
</tr>
<tr>
<td>Re-Weighting Frequency</td>
<td>Weightings will be monitored and changes will be made according to market conditions by BESA in consultation with market participants.</td>
</tr>
<tr>
<td>Calendar + Day</td>
<td>The Index is calculated every South African business day subject to the modified following day count convention.</td>
</tr>
</tbody>
</table>
Input Rates

The new index will use JIBOR as the input rates, which is a filtered average of the inter-bank deposit rates offered by designated contributor banks for maturities ranging from overnight to 1 year. The JIBOR construction methodology document can be downloaded from the BESA website1. The shorter rates are usually quite reliable and tend to accurately reflect market conditions at measurement time.

i. South African Benchmark Overnight rate
ii. 3 Month Johannesburg Inter-Bank Offer Rate (JIBOR)
iii. 6 Month JIBOR
iv. 9 Month JIBOR
v. 12 Month JIBOR

Steps for calculation of Overnight Deposit Sub-Index

**Step 1:** Source – South African Benchmark Overnight rate (SABOR)

**Step 2:** Calculate monthly return

\[ r_m = \frac{31}{365} \cdot y_t \]

Where:

- \( r_t \) – Expected monthly return on day \( t \)
- \( y_t \) – SABOR on day \( t \) (expressed as percentage)

**Step 3:** For valuation day \( t \), calculate average return over the past 31 business days

\[ A_t = \frac{\sum_{i=1}^{n} r_d}{n} \]

Where:

- \( A_t \) – Average return
- \( r_d \) – Monthly return assumed on day \( d \)
- \( n \) – Number of days = 31

**Step 4:** Calculate \( n_{bd} \), day between successive business days

\[ n_{bd} = d_t - d_{t-1} \]

Where:

- \( d_t \) – Valuation date
- \( d_{t-1} \) – Previous business day

**Step 5:** The return \( r_t \) over \( n_{bd} \) days

\[ r_t = \left( 1 + A_t \right)^{n_{bd}} - 1 \]

**Step 6:** The level of the sub-index \( I_t \), on valuation date \( t \) is

\[ I_t = I_{t-1} \cdot (1 + r_t) \]

1. http://www.bondexchange.co.za/besa/action/media/downloadfile?media_field=8679
Steps for calculation of a Sub-Index with n-Month Maximum Maturity

The following table lists the sub-indices with n-month maturity and the generating instruments:

<table>
<thead>
<tr>
<th>Sub-Indices</th>
<th>Instrument</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 Month</td>
<td>3 Month JIBOR</td>
</tr>
<tr>
<td>6 Month</td>
<td>6 Month JIBOR</td>
</tr>
<tr>
<td>9 Month</td>
<td>9 Month JIBOR</td>
</tr>
<tr>
<td>12 Month</td>
<td>12 Month JIBOR</td>
</tr>
</tbody>
</table>

On a given valuation date, paper is bought and held to maturity. The future cash flow is discounted daily using the BMM Curve. The change in present value over consecutive business days is measured as return. This methodology is applied until we maintain a portfolio such that paper is bought and return is measured every consecutive business day. The average of the return between business days for paper that matures on or after valuation day but before the maturity of paper bought on valuation date is calculated.

**Step 1:** On day \( d_t \), paper is bought and held to maturity. The future value factor of paper bought on \( d_t \) for maturity bond \( i \) is given by:

\[
FV_{t,i} = (1 + \frac{(d_m - d_t)}{365} \cdot y_i)
\]

Where
- \( d_m \) – Maturity of paper (\( x \) months in the future – modified following)
- \( d_t \) – Trade date
- \( y_i \) – JIBOR rate corresponding to maturity \( i \in \{3 \text{ Month, 6 Month, 9 Month, 12 Month}\} \); expressed as percentage.

**Discount Factor**

**Step 2:** The discount factor on valuation date \( t \) for maturity \( m \) is defined by:

\[
df_{t,m} = \frac{1}{(1 + y_{t,m} \cdot \frac{(m-t)}{365})}
\]

Where
- \( t \) – Valuation date
- \( m \) – Maturity date
- \( y_{t,m} \) – JIBAR rate on valuation date \( t \) for maturity \( m \); expressed as percentage

**Step 3:** For valuation day \( t \), the present value of paper \( i \) with maturity \( m \), bought on trade date \( t' \) is given by

\[
PV_{t,i} = df_{t,m} \cdot FV_{t'}
\]

Where:
- \( df_{t,m} \) – Discount factor on valuation date \( t \) for maturity \( m \)
- \( FV_{t'} \) – Redemption amount of paper bought on trade date \( t' \)

**Step 4:** The return is then measured by the ratio of the present value of paper on valuation day relative to valuation day \( t-1 \).

\[
r_t = \frac{PV_t}{PV_{t-1}} - 1
\]

Where
- \( r_t \) – Return on valuation day \( t \)
- \( PV_t \) – Present value of paper on valuation day \( t \)
- \( PV_{t-1} \) – Present value of paper on valuation day \( t-1 \)
**Step 5:** The average return is then calculated for a portfolio of instruments which mature on or after valuation date $t$ but before the maturity of the instrument bought on valuation date $t$

$$R_t = \frac{\sum_{i=1}^{d_t} r_{i,t}}{d_n}$$

Where:

- $R_t$ – Average daily return for a given portfolio on valuation date $t$
- $d_n$ – Number of instruments in current portfolio. This is equal to the number of consecutive business days over the averaging period. (Since paper is bought on trade days and held to $X$ months in the future (modified following), it is possible that more than one paper matures on a given business day)

The index on day $t$ for an $n$ month portfolio is:

$$I_t = I_{t-1} (R_t + 1)$$

**Composite Index**

A composite index is calculated from the weighted sum of the five indices.

<table>
<thead>
<tr>
<th>Sub-Index</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overnight Deposit</td>
<td>15 %</td>
</tr>
<tr>
<td>3 Month Sub-Index</td>
<td>25 %</td>
</tr>
<tr>
<td>6 Month Sub-Index</td>
<td>25 %</td>
</tr>
<tr>
<td>9 Month Sub-Index</td>
<td>15 %</td>
</tr>
<tr>
<td>12 Month Sub-Index</td>
<td>20 %</td>
</tr>
</tbody>
</table>

The weighted sum is calculated by:

$$I_{comp,t} = \sum_{i=1}^{5} w_i I_{i,t}$$

The above weights were determined by market participants and will be closely monitored and changed according to market conditions by BESA in consultation with market participants.

**Precision and Rounding**

All values are calculated and held to IEEE double precision (15 significant digits) except for the index values which will be published rounded to three decimal places.
Appendix

BESA Money Market Yield Curve Calculations

1. Introduction

BESA has built a new money market curve which will be used for discounting cash flows of the Money Market Index as well as serve as a standardised curve to be used for pricing mark-to-market portfolios on a daily basis. To find the discount factors of a par rate yield curve, a technique called bootstrapping is followed. The bootstrap technique is used to determine discount factors from market rates of financial instruments that pay intermediate coupons before final maturity. In this manner a consistent set of discount factors will be determined that will finally result into the same yields to maturity as stated by the market rates.

2. Data Collection

The rates used for the yield curve construction will be made available at request.

3. Calculation of Discount Factors

Cash flows at different times are not directly comparable. To compare cash flows, it is assumed that receiving a future payment is equivalent to receiving a smaller payment today and investing it in a zero coupon instrument.

The present value of a future cash flow is defined by:

\[ PV = \frac{FV}{(1 + r \alpha)} \]

Where :
- \( PV \) = present value of cash flow
- \( FV \) = future value of cash flow
- \( r \) = interest rate
- \( \alpha \) = day count fraction

The day count fraction is the ratio of number of days in period under consideration over number of days in year depending upon the day count convention.

The discount factor is defined by:

\[ df = \frac{1}{(1 + r \alpha)} \]

Consider a single instrument with present value \( N \), discount factors \( df_i \) to \( df_n \) and cash flows \( C_1 \) to \( C_n \) such that:

\[ N = C_1 df_1 + C_2 df_2 + \ldots + C_{n-1} df_{n-1} + \left( N + C_n \right) df_n \]

which includes a repayment of the notional amount \( N \) at maturity \( n \).

The equation cannot be solved uniquely and therefore a system of equations can be developed to solve the discount factors uniquely. This is achieved by solving the discount factor of the next maturity in terms of the preceding maturity’s discount factor.

Define:
- \( r^k \) = coupon rate for the \( k^{th} \) maturity.
- \( \gamma \) = assume coupons are paid semi-annually.
- \( y \) = number of days per year.
- \( d_{ki} \) = number of interest days in coupon period \( i \) for the \( k^{th} \) maturity

Then the \( i^{th} \) cash flow for the \( k^{th} \) maturity is given by:

\[ C_{k,i} = \frac{r_k}{2} \]

The discount factors for maturities that pay no intermediate coupons is given by:

\[ df_k = \frac{N}{(N + r_k d_{k,i} / y)} \]

While the discount factors for maturities that pay intermediate coupons is bootstrapped by means of the following equation:

\[ df_k = \frac{\left( N - \sum_{i=1}^{k-1} C_{k,i} df_i \right)}{(N + C_{k,k})} \]
4. Interpolation

The valuation of most products will require the calculation of discount factors for dates in between the standard yield curve dates. The following method uses linear interpolation of continuous compounded zero-coupon rates \( z_{cc} \), i.e. the discreetly compounded zero-coupon rates are converted to continuous-compounding equivalents:

\[
1 + Y = \left( \frac{1}{df} \right) = e^{\frac{z_{cc} \times df}{y}}
\]

Therefore,

\[
df = e^{-z_{cc} \times \frac{d}{y}}
\]

And

\[
z_{cc} = -\frac{y}{d} \times \text{Ln}(df)
\]

If we have a pair of zero rates \( z_1 \) and \( z_2 \) with known discount factors \( df_1 \) and \( df_2 \) we can solve for any other intermediate point \( i \):

\[
z_{cc} = -\frac{y}{d} \times \text{Ln}(df_i)
\]

Linear interpolation on the continuous compounded zero rates leads to the following formula:

\[
df_i = df_1 \times df_2
\]
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