JSE ASSA Fixed Income Indices | September 2010

By the Johannesburg Stock Exchange in conjunction with the Actuarial Society of South Africa

JOHANNESBURG STOCK EXCHANGE
Interest Rates

This document replaces the original index document, BEASSA TOTAL RETURN INDICES 2000, August 2000. The majority of the content has been incorporated into this document for the purposes of continuity.

Contents

1 INTRODUCTION

This paper covers the calculation, uses and application of the JSE ASSA Fixed Income Indices; this consists of the Vanilla Index, the Inflation-Linked Index and all their respective sub indices. This paper makes reference to the bond pricing formula and older indices such as the clean price index and the interest yield index. Information on these specific subjects is contained in their respective papers which are all available through the JSE website or by contacting the JSE interest rate division directly.

2 THE INDICES AND THEIR MANAGEMENT

The JSE ASSA Vanilla Index known as the "**ALBI**" is a composite index containing the top 20 vanilla bonds ranked dually by liquidity and market capitalisation (see Selection of Constituents). The **ALBI** is split into two sub-indices. The "**GOVI"** index contains the top 10 government bonds within the **ALBI** index. The liquidity and credit characteristic of government bonds issued by the Department of Finance is fundamentally different from other bonds in the market and this is, in part, the reason for this sub-division. The remainder of the bonds in the **ALBI** index will fall into the "**OTHI**" index.

The latest addition to the suite of JSE indices are the Inflation-Linked Indices which track the general levels of bonds whose returns are linked to the Consumer Price Index, or "CPI", published monthly by Statistics South Africa in document P0141. The CPI figure used is the Headline CPI for all urban areas as this is the figure on which most inflation-linked bonds are adjusted and also happens to be the figure used for the South African Reserve Bank's inflation targeting.

The Composite Inflation-Linked Index is called the "**CILI**" and is split into three sub-indices to reflect bonds issued by Government, State Owned Enterprises and Corporates which will be called the "**IGOV**", "**ISOE**" and "**ICORP**" indices respectively. Other than the inherent credit quality differences between these broad categories they act as loose subdivisions of market liquidity as well.

To be included in the Government category a bond must be issued by the Republic of South Africa, the State Owned Enterprises category holds any other bond that boasts a government guarantee and the remainder of the bonds will fall into the Corporate category.

In addition, both the **ALBI** and the **CILI** are split into four sub-indices based on term to maturity forming the 1-3 year, 3-7 year, 7-12 year and 12+ year sub-indices.

The following values are now published daily by the JSE. "Old" indicates values that have already been published by the JSE for some time and "New" indicates values that are new to this revision of the document.

SECTOR		CLEAN PRICE INDEX	INTEREST YIELD INDEX	TOTAL RETURN INDEX	MODIFIED DURATION & CONVEXITY
ALBI		Old	Old	Old	Old
CILI		New	New	New	New
LIQUIDITY SPLITS:					
ALBI	GOVI	Old	Old	Old	Old
	OTHI	Old	Old	Old	Old
CILI	IGOV	New	New	New	New
	ISOE	New	New	New	New
	ICORP	New	New	New	New
TERM SPLITS:					
ALBI	1-3 years	Old	Old	Old	Old
	3-7 years	Old	Old	Old	Old
	7-12 years	Old	Old	Old	Old
	12+ years	Old	Old	Old	Old
CILI	1-3 years	New	New	New	New
	3-7 years	New	New	New	New
	7-12 years	New	New	New	New
	12+ years	New	New	New	New

3 SELECTION OF CONSTITUENTS

3.1 Overview

3.1.1 Composites

The constituents of the indices will be re-selected once a quarter. The re-selection will be effective from the day following the expiry of standardised OTC bond options and the JSE's bond futures contracts.[1]

The selection process will be performed early in the previous month, so that adequate notice of changes may be given. The selection for the indices will be based upon the average market capitalisation and liquidity (clean consideration turnover). These values are averaged over the twelve preceding months which is called the "Averaging Period". Bonds newly listed within an Averaging Period will have their liquidity and market capitalisation averaged only over the months for which they have been listed, and hence will not be handicapped.

Only conventional listed bonds, with a remaining life greater than one year throughout the quarter, will be eligible. Bonds with a term less than one year are excluded from all indices as they tend to be illiquid and behave more like money market instruments.

The selection process will select, uniquely, the first *N* bonds according to a dual ranking scheme based on the liquidity and market capitalisation. These *N* bonds are then the constituents of the composite indices namely the Vanilla Index (ALBI) and the Inflation-Linked Index (CILI).[2]

[1]Standardised OTC options and bond futures expire on the first Thursday of February, May, August and November. If this first Thursday is a public holiday, the expiry is on the previous trading day. Re-basings are assumed to happen at the close on the first trading day preceding the effective date of the change.

[2] Initially the **ALBI** index will contain 20 bonds and the **CILI** will contain 15

3.1.2 Sub-Indices

The **GOVI** index will contain all bonds issued by the Republic of South Africa that fall into the top 10 positions of the **ALBI** according to the dual ranking scheme. The remainder of the **ALBI** by definition are constituents of the **OTHI** Index.

The sub-indices for the inflation-linked composite are based solely on the issuing body creating Government, State Owned Enterprise and Corporate sub-indices. These will be called the "**IGOV**", "**ISOE**" and "**ICORP**" respectively. **IGOV** bonds must be issued by the Republic of South Africa, **ISOE** bonds must be government guaranteed but not issues by the Republic of South Africa and all other bonds fall into the **ICORP** sub-index.

The selected bonds will be weighted in their various indices according to their nominal amounts in issue as on the last day of the Averaging Period. The bonds within the index are re-weighted in January, March, April, June, July, September, October and December to compensate for changes of the nominal amount in issue. Re-constitution of the entire index is done at the end of every quarter for February, May, August and November, except possibly in the case of an Extraordinary Event (see Section 3.3). The reweighting will be effective on the Friday following the first Thursday of the reweighting month and the reconstitution will be effective at 12noon on the first Thursday of the reconstitution month.

Figures for market capitalisation and liquidity for the re-selections will be calculated by the JSE's market statistics department. The JSE will be the sole arbiter as to the correctness of these figures.

3.2 Selection

3.2.1 Eligible Bonds

To be eligible for selection, a bond must:

- Be listed on the JSE and settled electronically;
- For the Vanilla Index:
  - Be conventional or "plain vanilla", meaning that it has a fixed (even if zero) semi-annual coupon

- For the Inflation-Linked Index:
  - Be a conventional CPI instrument which has a fixed (even if zero) semi-annual coupon inflated by the four month lagged headline Consumer Price Index for all urban areas (see Section 4.4.4 for the formula). This is in line with current market practices for inflation-linked bonds and the specifications laid out by the International Swaps and Derivatives Association (ISDA)

- Have EITHER a single redemption date; OR, in the case of bonds with multiple redemption dates, be priced in the market according to the "mid-redemption" convention; and a redemption date (or mid-redemption date) which coincides with a coupon date.

- Have a remaining term greater than 1 year over the entire quarterly period for which the selection is being made.

3.2.2 Averaging Period

The selections will be based upon average size of the issue and liquidity for the twelve month period preceding the selection date. The Averaging Periods are:

February re-constitution: the January of the previous year to the previous December.
May re-constitution: the April of the previous year to the previous March.
August re-constitution: the July of the previous year to the previous June.
November re-constitution: the October of the previous year to the previous September.

The averages are calculated over the months in the Averaging Period; or, in the case of new issues, over the months in the Averaging Period for which they have been issued. The quarterly re-constitution will coincide with bond option and futures expiry dates to encourage the creation of new derivatives and to improve the efficacy of hedging.

For the re-weightings, the nominal amount in issue is taken to be the value at the end of the month, two months prior to the month in which the new re-weighting applies. For example, the values applicable for the month of March will be the nominal amount in issue as of the end of January.

3.2.3 Market Capitalisation

The market capitalisation is required for each eligible bond as of the end of each month in the Averaging Period. The market capitalisation of a bond is defined as the product of its nominal in issue and its clean price.[3] The nominal in issue (as distinguished from the nominal listed) will be as at the end of the month, and the price will be calculated at the bond's closing yield to maturity for the month. For inflation linked bonds, the inflation adjusted clean price based on the real yield to maturity is used.

3.2.5 Liquidity

The liquidity of a bond is its secondary market turnover. Trades included when considering the JSE ASSA Indices are outright purchases and sales, of standard and odd lots, for settlement on the standard settlement day ("t+3") or on any non-standard day. Both legs of carries (also known as buy/sell-backs), Reserve Bank's repos and option exercises are excluded. This applies to both Vanilla and Inflation-Linked Indices.

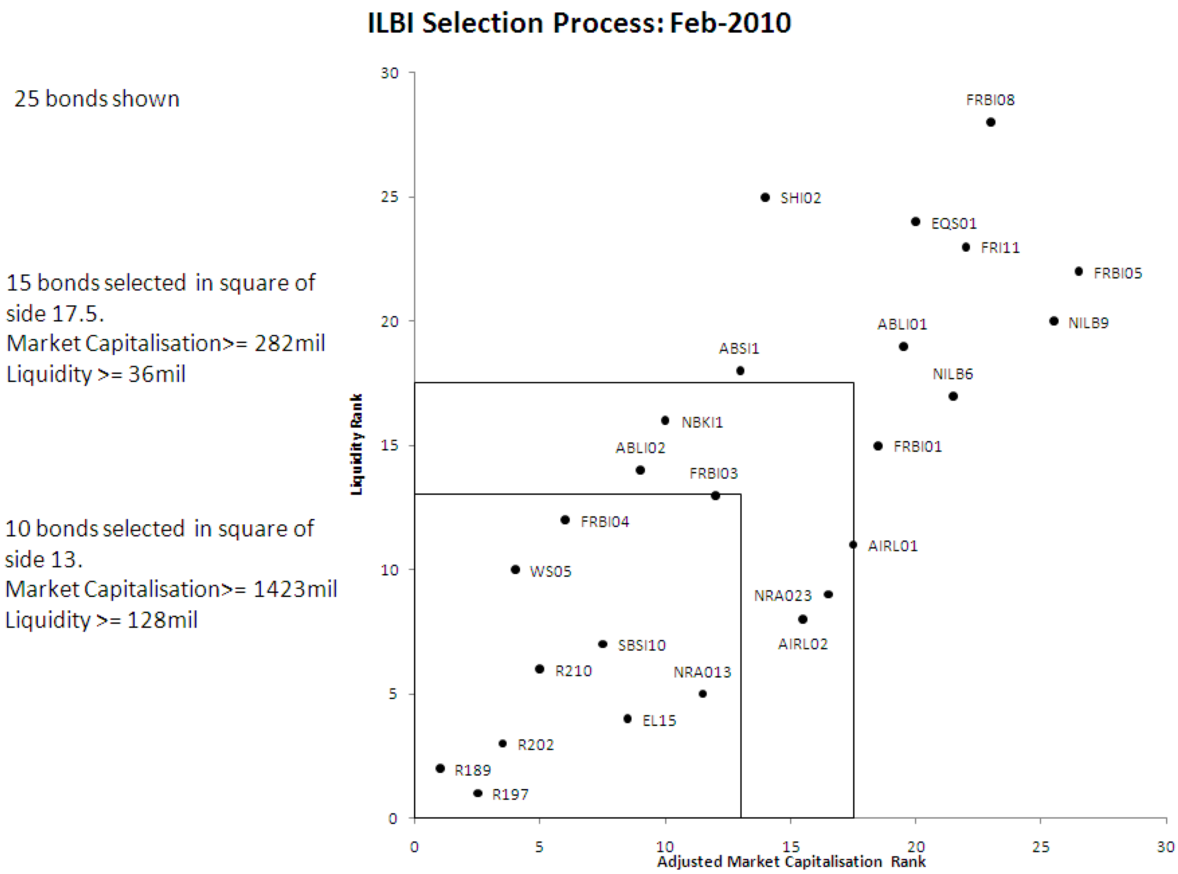
The value traded is given by the clean consideration of each trade. Each day's value traded will be aggregated to give a total figure for each month in the Averaging Period

3.2.6 Dual Ranking: an Illustration

Bonds are selected under the dual criteria of a market capitalisation measure and a liquidity measure. The dual ranking method ascribes equal importance to the rankings of bonds under the criteria. This method has the virtue, above all, of simplicity.

The figure on the next page gives an example. It shows a scatter diagram of the rankings of the inflation-linked bonds for the **CILI** index. Bonds are selected by overlaying notional squares (of which two are shown) who's lower left hand corners are anchored at the origin. The size of the square is increased until it covers exactly the number of bonds required.

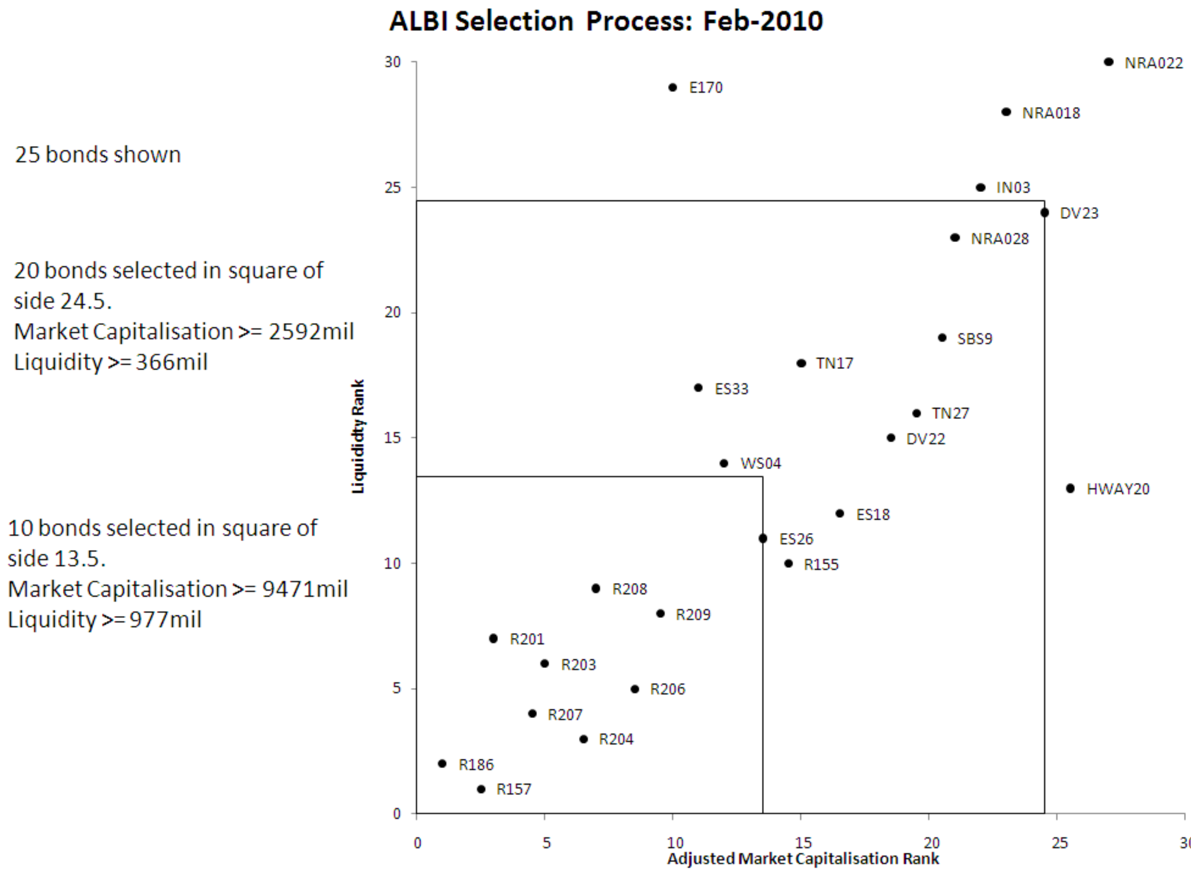
[3] Because prices are per R100 nominal, the result must be divided by 100.



The diagram shows that, for the period of this example, the inflation-linked bond with the highest market capitalisation was the R189, while the most liquid bond was the R197. It is also interesting to note the relatively high liquidity of the NRA013, compared to its relatively low market capitalisation, while the position is reversed for the WS05.

The square containing 15 bonds shows how an inflation-linked composite might be selected for the February 2010 re-constitution. The square of sides 13 illustrates the bonds that would be included if N was set equal to 10. The selection of the **IGOV** index cannot be shown in this way; because its constituents are defined by an external criterion (bonds must be issued by the government). Initially the **IGOV** index is expected to be comprised of the four government bonds shown in this square, these are also the only inflation-linked bonds issued by the government at time of writing.

We use the same process for selecting the constituents of the **ALBI** index. On the next page is an illustration of the selection process for the February 2010 re-constitution of the **ALBI**.



Once a given number of bonds has been selected for an index, the smallest market capitalisation and liquidity of all the selected bonds can be determined (these figures may well belong to different bonds). It is then true that:

- All selected bonds have both market capitalisation and liquidity greater than or equal to these minima;
- No unselected bonds have both greater.

3.2.7 Selection Algorithm

This section provides an algorithm to select bonds with the characteristics described above.

1. Rank the bonds by average market capitalisation over the Averaging Period, attaching to each bond its "Market Capitalisation Rank". Ties (a highly unlikely event) are broken alphabetically by bond code.
2. Rank the bonds by average liquidity over the Averaging Period, attaching to each bond its Liquidity Rank". Ties are broken alphabetically-in-reverse by bond code.
3. The Dual Rank of a bond is the greater of its Market Capitalisation Rank and its Liquidity Rank. Eventually bonds are going to be selected in ascending order of Dual Rank. However, ties of Dual Rank are likely, even common. They are broken by Market Capitalisation Rank as determined in point one above. This can be achieved by adding 0.5 to the Market Capitalisation Rank of any bond whose Market Capitalisation Rank is greater than or equal to its Liquidity Rank. The result is called the "Adjusted Market Capitalisation Rank".
4. The Dual Rank of a bond is then the greater of its Adjusted Market Capitalisation Rank and its Liquidity Rank.
5. List bonds in ascending order of Dual Ranks. The first N bonds so listed are selected.

3.3 Extraordinary Events

In certain circumstances it might be necessary to consider the re-selection of constituents during a quarterly period. These circumstances could include (but are not limited to):

1. The early redemption or repurchase by the issuer of a large proportion of a bond's outstanding issue.
2. The issue of a new bond which is too large or important to have to wait for the next quarterly re-selection. This is based on the presumption that the bond will remain in the index.
3. The unbundling of a bond, as for example happened to the R153, when the issuer offered an optional conversion into three new bonds.
4. In other extraordinary cases, the JSE ASSA Index Committee will be convened to decide on a course of action. The decisions to be taken will include:
    i. The new constituents of the sectors (which could however remain the same);
    ii. Their new weightings (which could also be unchanged);
    iii. The effective date for the change (if any);
    iv. The amount of notice required (if applicable).
5. Any changes so introduced will be effective only until the next quarterly re-selection, at which the standard rules will again apply. In making these decisions, the committee will follow these guidelines:
6. The overriding principle of the indices is that they track a portfolio of the constituent bonds in their given proportions. Hence any changes must be theoretically replicable by fund managers[4].
7. Liquidity is an issue. It may sometimes be unreasonable to expect fund managers to be able to move large amounts of stock. On the other hand, the very requirement to do so may create its own liquidity.

The committee will recognise that these specifications are unable to foresee all circumstances, and hence its discretion will be needed from time to time. However, these actions will form precedents, which will inform the committee's subsequent deliberations.

The committee will communicate its decisions as soon as possible through JSE Members' Notices and press releases, where appropriate, with back-up information on the JSE website. Ample notice will be given.

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[4] Assuming liquidity, no transaction cost and no market impact.

4 INDEX CALCULATIONS

4.1 Overview

The prime design criterion of the JSE ASSA Fixed Income Indices is replicability. This means that a fund manager should theoretically be able to match the performance of the index by passively investing in a replicating portfolio.

Therefore, the specifications which follow are cast in terms of a "Reference Portfolio", rather than the "basing constant, *k*" which is often used in specifying indices. The two approaches are, however, mathematically equivalent.

The Reference Portfolio contains two classes of asset: bonds, and (from time to time) the rights to receive coupons. The nominal amounts of the bonds are proportional to their weights, which are their total nominal amounts in issue, updated monthly. This is the so-called "nominal weighting scheme". Coupons vest on holdings of bonds on the days they go ex-coupon. They are received some time (usually 10 days) later and re-invested in the portfolio.

The Bond Portion of the portfolio is valued at all-in prices, using the appropriate market yields to maturity for the valuation date, for settlement on the standard settlement date ("t+3"). Coupons are valued by discounting them from receipt date to standard settlement date, at the yields to maturity of their parent bonds.

Bond and coupon values for the settlement date are then further discounted, at the yields to maturity of their respective bonds, to the valuation date.

Changes in the holdings of bonds in the Reference Portfolio are referred to as "re-basings" and occur for four reasons:

1. Re-investments. Coupons are re-invested upon receipt, in all bonds in the portfolio, according to their weights.
2. Re-constitutions. For the months of February, May, August and November the constituents of the portfolio will be re-selected and their weights updated. Purchases and sales will be required to mirror the new weightings.
3. Re-weightings. For the months of January, March, April, June, July, September, October and December the index is re-weighted to reflect changes in the nominal amount of bonds in issue.
4. Term sub-sector changes. Bonds in both the ALBI and CILI are allocated to the four term sub-sectors according as to whether their remaining life is greater than the lower limit, and less than or equal to the higher limit for the sub-sector. Hence, for example, a bond will be moved from the 3-7 year to the 1-3 year sub-sector on the first day for which its remaining life is less than or equal to three calendar years. A bond moving sector in this way must be sold from one reference portfolio and bought into the other one on the first preceding trading day. It will maintain the same market capitalisation weighting; and will leave behind any ex-coupons in its former portfolio.

Re-basings are assumed to happen instantaneously at a fixed time (the "re-basing time") on the last trading day before the change becomes effective. They take place at the yields to maturity determined by the JSE to hold at that time ("re-basing yields"). This would normally be at 16h30 each day, using the mark-to-market yields to maturity determined by the JSE at that time.[5]

A portfolio which is re-based at the re-basing yields should track the index. In order to make this requirement reasonable, it is recognised that JSE may have to introduce an averaging process to determine re-basing yields on days when substantial re-basings are due.

[5] It may be necessary to use a different time on days when futures contracts expire, if they have a different expiry time. Currently the 12h00 mark to market is used for re-constitutions as this is when the future contracts expire.

4.2 Performance Measurement

The utility of the JSE ASSA Indices arises out of the ease with which they allow performance to be measured between any two dates. If the index on the first date is I_0 and on the second date is I_1 , then the performance between these dates is given by:

$$P = \frac{I_1}{I_0} - 1$$

(1)

This formula measures performance as a decimal (e.g. the decimal 0.10 means 10%) and gives it for the period between the two dates, without any annualisation. This is the convention for periods of less than one year.

For periods of greater than one year, it is conventional to annualise the performance as follows. Let the first date be t_0 and the second be t_1 , both measured as days from some base date (so that the number of days between them can be found by subtraction). Then, the annualised performance of the index between the two dates is:

$$P_A = \left(\frac{I_1}{I_0}\right)^{\frac{365}{t_1 - t_0}} - 1$$

(2)

This measure of performance is still a decimal, and is in nominal annual compounded annually (NACA) units. These units allow direct comparison with performance figures for other markets, for example the JSE.

It may sometimes be desirable to state performance in nominal annual compounded semi-annually (NACS) units. This allows direct comparison with bond yields, which are quoted in the same units. This measure is given by:

$$P_S = \left(\left(\frac{I_1}{I_0}\right)^{\frac{\frac{365}{2}}{t_1 - t_0}} - 1\right) \times 2$$

(3)

4.3 Applications of the Index Value

The characteristics described above make the JSE ASSA Fixed Income Indices essential investment management tools for asset managers. They have the following specific uses.

4.3.1 A barometer for daily movements in the bond market

The Vanilla Indices are comprised of "vanilla" bonds from across the full range of maturities in the bond market and are a useful summary measure of the daily movement in the bond market while the Inflation-Linked Indices provides a summary measure in the inflation-linked bond space. This is preferable to monitoring the yield or price movement on a single benchmark bond which will bear very specific supply and demand, credit, term and liquidity characteristics which is not necessarily comparable to the market as a whole.

4.3.2 A benchmark for measuring portfolio performance

The JSE ASSA Indices accurately measure the performance of the bond market over time. Investors can compare the performance of their bond portfolios against the market as measured by the appropriate index. Comparison with the various sub-indices allows investors to assess how successful their asset selection has been. This can be illustrated as follows:

JSE ASSA Sub-Indices (last 12 months)		
	Performance	Duration
ALBI	8.93%	5.96
GOVI	8.76%	5.98
OTHI	9.77%	5.88
CILI	6.98%	7.79
IGOV	6.62%	8.41
ISOE	6.84%	5.44
ICORP	11.25%	3.07

Hypothetical Bond Portfolio Returns (last 12 months)			
	Performance	Duration	Proportion of Portfolio
GOVI	9.52%	6.73	50%
IGOV	7.03%	9.17	50%
TOTAL	8.28%	7.95	100%

The comparison shows that while the hypothetical bond portfolio has outperformed the indices, it seems that this performance may be attributable to the longer duration of the portfolio. This analysis can be done by using the term splits rather than the credit splits in order to analyse which terms are responsible for the excess performance.

4.3.3 A basis for funds tracking the bond market

The JSE ASSA Fixed Income Indices can be used as a target index by investors requiring passive investment management of their bond portfolio. The indices have been designed to ensure replicability through the liquidity and market capitalisation criteria for constituents.

4.3.4 Analysing sub-sectors of the market

The sub-indices allow analysis of the market by term and by liquidity for both the vanilla bond market and the inflation-linked bond market. These divisions ensure that a portfolio benchmarked against these sub-indices does not suffer from differences in the term structure or credit spread of the composite indices.

4.4 Calculation of the JSE ASSA Indices

The description which follows is designed for programmers writing to these specifications, rather than for immediate edification of the reader. Therefore some prefatory explanation may be in order.

Sections 4.4.1 to 4.4.6 define the basic concepts, from the time structure (valuation and settlement dates), through the nomenclature for bonds and their weights, and onto the initialisation of an index at its inception. The last of these sections defines the discount factors to be used between settlement and valuation date.

Section 4.4.7 describes the bond portion of the portfolio, in terms of the nominal amounts of each bond, and specifies how to value it.

Sections 4.4.8 to 4.4.11 deal with ex-coupons. First the ex-period is defined and then the quantification and valuation of ex-coupons during this period are specified. Lastly, the value of coupons being re-invested on any day is given.

Section 4.4.12 gives the central result: the calculation of the Nominal Factor on any day, to reflect coupons being re-invested and any change in weights. The same formula can be used even if there are no re-investments or weight changes.

Finally, Section 4.4.13 gives the total portfolio value on any day, and shows how the index value is found from it.

4.4.1 Time Structure

Consider the calculation of the index value at a certain time on a certain date, the "Valuation Date", *t*.[6] The Valuation Date may be any day, and not just trading days. The calculation time is not parameterised, but is stated explicitly where it is not obvious from the context. For any date, t, define its settlement date, s, as follows:

If t is a trading day, s is the date of its settlement day according to the then prevailing JSE practice. Currently bonds settle on "t+3" using the following good business day convention.

If t is a non-trading day, s is deemed to be the settlement day of the first preceding trading day.

Yields to maturity applicable to any calculation, unless explicitly stated, will be those prevailing at the time. For calculations on non-trading days the mark-to-market yields to maturity of the first preceding trading day will be used.[7] Nominal yields will be used for vanilla bonds and real yields for inflation-linked bonds.

4.4.2 Bonds

The constituent bonds of the index being calculated are referred to by the subscript *i*, or as "bond i", where *i* ranges from 1 to *N*, the number of constituent bonds. All sums (∑) below are from *i=1* to *N*. *N* obviously differs between sectors, and hence the description below is to be taken as referring to the calculation for a single sector only.

[6] Dates such as t are expressed as the number of days from a fixed base date so that simple arithmetic can be performed upon them.

[7] Yields are assumed to be given as percentages, so that a yield of 16.52% is the number 16.52. Division by 100, where necessary, is explicit in the calculations.

4.4.3 Weights

The weights of the bonds in the Index are given by:

$$w_{i,t} \tag{4}$$

where the subscript *i* refers to bond *i* and the subscript *t* indicates the date.

The weights are the Rand nominal amounts issued of each bond, measured in suitable units (e.g. R millions). Note that, although these specifications allow for the weights to change every day, in general they will change only once a month. On days when there is no change, the specifications can be followed with the unchanged values of the weights and the results will be correct.

On days when re-weighting, re-constitutions or term sub-sector changes take place, weights for some bonds could be zero either before or after the change. The specifications below allow for this. In fact, any number of extra bonds can be included in the N bonds, and as long as their weights are zero, they will not affect the results.

4.4.4 Inflation Factor

An inflation index ratio is needed to adjust certain values when calculating the Inflation-Linked Indices. While the all-in price of the bond published by the JSE is already adjusted for inflation, there are two instances where the $CPI(j)$ index ratio is needed. Firstly, to create a nominal discount factor from a real discount factor and secondly, to calculate the actual cash flow of an ex-coupon.

The CPI index used in the calculation of the index ratio is the headline CPI for all urban areas published monthly by Statistics South Africa in document P0141. [8]

$CPI(j)$ or the value of the CPI index ratio at a given time *j* is calculated as per ISDA specifications as follows:

$$CPI(j) = \frac{\frac{m-d+1}{m} CPI_{M-4} + \frac{d-1}{m} CPI_{M-3}}{Base\ CPI} \tag{5}$$

Where

j is the date for which the CPI index ratio is needed

d is the calendar day corresponding to *j*

M is the calendar month corresponding to *j*

m is the number of days in the calendar month *M*

CPI_X is the published CPI index value that applies to the first day of the calendar month *X*

$Base\ CPI$ is the CPI index value for the base date, this is usually the issue date of the bond.

CPI(j) is the CPI index ratio applicable to date *j*

[8] This index is subject to change and rebasing by Statistics South Africa. If the aforementioned index is not available, the relevant substitute index must be used.

4.4.5 Initialisation

An index is created as at the end of date t_0. Its initial value is I_{t0}, as at the close of day t_0. The Rand value of the Reference Portfolio at the same time is Z_{t0}. Indices are usually launched with an initial value of 100, although there is no theoretical reason why other values cannot be used. The value of the Reference Portfolio "cancels out" in the calculations, and therefore its initial value can be almost anything. A convenient value is R100m, for then the portfolio value (in R millions) and the index value (if initially 100) are always the same number. The base date will be the 1st of July 2000 for the **ALBI** and the 1st February 2007 for the **CILI**. All the sub indices of the **CILI** will begin on the 1st of February 2010 with an index value of 100 but the **IGOV** will be backdated to the 5th of February 2010. This ensures that the index levels are somewhat comparable as their levels are identical on the 1st of February 2010

4.4.6 Valuation Date Discount Factors

The calculations for day *t* start with the pricing of the constituent bonds in the Reference Portfolio for settlement on *s.* Similarly, ex-coupons (if any) will be discounted from their receipt date to give their value on *s*.

These values for *s* will then be further discounted from *s* to the valuation date, *t*. The discounting of each value (i.e. bond price or ex-coupon belonging to a bond) will be at each bond's yield to maturity, which will be a real yield, including an inflation adjustment, for inflation-linked bonds.

The effect of this further discounting will often be the same as if the bond itself had been valued for settlement on *t* and not on *s*, using the same yield to maturity.[9] In order to make this statement as true as possible, the discounting method applied from *s* to *t* must be consistent with that in the Bond Pricing Formula.[10] The specifications below describe the calculation of discount factors, $D_{i,t}$, for bond *i* on day *t*, which fulfil this condition.

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[9] Results will differ only where valuation for t is cum, while that for s is ex.

[10] Specifications for the Bond Pricing Formula are available from the JSE in a document of that name, as hard copy or on their website.

The inputs are:

t valuation date

s the settlement date for t (see 4.4.1)

$c_{i,t}$ the first coupon payment date for bond i which is on or after t

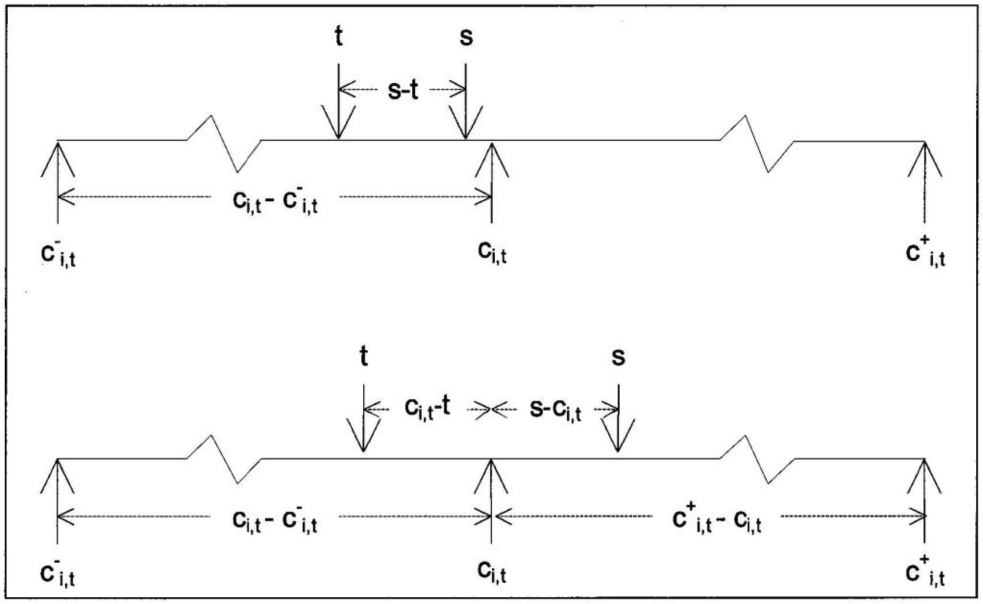
$c^-_{i,t}$ the previous coupon payment date for bond i which is before $c_{i,t}$

$c^+_{i,t}$ the following coupon payment date for bond i which is after $c_{i,t}$

$Y_{i,t}$ the yield to maturity of bond i applicable to the valuation. This figure will be the nominal yield to maturity for vanilla bonds and the real yield to maturity for inflation-linked bonds

The first step is to determine the exact number of half-years, $H_{i,t}$, over which discounting is performed for bond i on day t. Under the Bond Pricing Formula, a "half-year" is deemed to be the period between the last and next coupon dates, and varies in its number of days. This means that the period from s to t, even though a fixed number of days, varies when measured in half-years and is a function of both the bond and the date.

If the period between t and s does not contain a coupon payment date, then $H_{i,t}$ is simply the number of days from t to s divided by the number of days in the current coupon period. However, if a coupon payment date does intervene, we have to allow for two coupon periods of different lengths. These two cases are illustrated below:



Hence

$$H_{i,t} = \frac{s-t}{c_{i,t} - c^-_{i,t}} \qquad if\ c_{i,t} \geq s$$

$$= \frac{s - c_{i,t}}{c^+_{i,t} - c_{i,t}} + \frac{c_{i,t} - t}{c_{i,t} - c^-_{i,t}} \quad if\ c_{i,t} < s$$

(6)

Given $H_{i,t}$, the discount factor is:

$$D_{i,t} = \left(\frac{1}{1 + Y_{i,t}/200}\right)^{H_{i,t}} If\ the\ bond\ is\ a\ \boldsymbol{Vanilla}\ Bond$$

$$D_{i,t} = \left(\frac{1}{1 + Y_{i,t}/200}\right)^{H_{i,t}} \times \frac{CPI(\text{t})}{CPI(\text{s})}\ If\ the\ bond\ is\ an\ \boldsymbol{Inflation\ Linked}\ Bond$$

(7)

4.4.7 The Bond Portion of the Portfolio

The Reference Portfolio is divided into two portions: the Bond Portion and the Ex-Coupon Portion. This section deals with the former.

The Bond Portion of the Portfolio is a portfolio of bonds *i* held in nominal amounts $N_{i,t}$ on day *t*. The nominal amount of Bond *i* on day *t* is proportional to that bond's weight on the same day:

$$N_{i,t} = K_t \times w_{i,t} \qquad \forall i, t \tag{8}$$

The proportionality constant K_t is called the Nominal Factor. It is subject to change day by day, as described in Section 4.4.12 below. However, its initial value is given by the requirement that the portfolio's initial value is Z_{t_0}:

$$K_{t_0} = \frac{Z_{t_0}}{\sum w_{i,t_0} \times P_{i,t_0} \times D_{i,t_0}} \tag{9}$$

where P_{i,t_0} is the all-in price of Bond *i* at its re-basing yield for day t_0, for settlement on t_0's standard settlement day, and w_{i,t_0} are the weights applicable to t_0.[11]

The value of the Bond Portion at any time on any day t is given by:

$$B_t = \sum N_{i,t} \times P_{i,t} \times D_{i,t} \tag{10}$$

where the $P_{i,t}$ are the all-in prices of the bonds *i*, for settlement on day *s*, at the yields to maturity prevailing at the time. For the inflation-linked indices, the all-in prices provided by the JSE will already

[11] Note that prices as defined in Bond Pricing Formula are per R100 nominal, and must be divided by 100 before being used here.

include the CPI index ratio adjustment (defined in section 4.4.4) to the price and hence the nominal amounts will be inflated appropriately.

4.4.8 The Ex-Period

The Ex-Period of a bond is the period, occurring before each of its coupon payment dates, when it is trading for settlement ex-coupon.

The Ex-Period begins on the first day on which the bond is trading for ex-settlement, i.e. it begins on the first *trading* day whose settlement day is *on or after* the bond's books-closed day.

It ends on the first day on which the coupon (having been received) can be re-invested, i.e. it ends on the first *trading* day whose settlement day is *on or after* the bond's coupon payment day. On this day, the ex-period ends at the re-basing time for the day.

4.4.9 The Ex-Coupon Portion of the Portfolio

The Ex-Coupon Portion of the Portfolio as at day t is the total Rand amount of coupons which have vested in the Portfolio (based upon holdings in the bonds at the start of their Ex-Periods) but have not yet been received and re-invested in the Portfolio.

The ex-coupon relating to bond *i* on day *t* can be defined as:

$$X_{i,t} = N_{i,t'} \times \frac{g_i}{200} \quad \mathit{If\ the\ }\boldsymbol{Vanilla}\mathit{\ bond\ is\ in\ the\ ExPeriod}$$

$$X_{i,t} = N_{i,t'} \times \frac{g_i}{200} \times CPI(\mathrm{c_{i,t}}) \ \mathit{If\ the\ }\boldsymbol{Inflation\ Linked}\mathit{\ bond\ is\ in\ the\ ExPeriod}$$

$$X_{i,t} = 0 \qquad \mathit{Otherwise}$$

(11)

Where

$N_{i,t'}$ is the nominal amount of the bond in the portfolio on the first day, t', of its Ex-Period (and before any re-basings for that day);[12]

[12] Note that, on launch of a new index, the fact that a bond may be ex is ignored, so that all ex-coupons are zero.

g_i is the coupon rate of bond *i*, expressed as a percentage. This will be a nominal coupon rate for vanilla bonds and a real coupon rate for inflation-linked bonds

$CPI(c_{i,t})$ is the CPI index ratio for the coupon payment date $c_{i,t}$ as illustrated below.

$$CPI(c_{i,t}) = \frac{\frac{m-d+1}{m} CPI_{M-4} + \frac{d-1}{m} CPI_{M-3}}{Base\ CPI}$$

Where

$c_{i,t}$ is the date for which the CPI index ratio is needed, in this instance, the next coupon payment date for bond *i* at valuation date *t*

d is the calendar day corresponding to $c_{i,t}$

M is the calendar month corresponding to $c_{i,t}$

m is the number of days in the calendar month $c_{i,t}$

CPI_X is the published CPI index value for the first day of calendar month *X*

$Base\ CPI$ is the CPI index value for the base date of bond *i*, this is usually the issue date of the bond.

$CPI(c_{i,t})$ is the CPI index ratio applicable to date $c_{i,t}$

4.4.10 The Value of the Ex-Coupon Portion

The value of each bond's ex-coupon on any day *t* in its Ex-Period, is its present value, discounted first from coupon payment date to *s*, and then from *s* to *t*:

$$V_{i,t} = X_{i,t} \times D_{i,t} \times \left(\frac{1}{1 + \frac{Y_{i,t}}{200}} \right)^{\frac{\max(c_i - s, 0)}{c_i - c_i^-}}$$

$$\mathit{If\ the\ bond\ is\ a}\ \boldsymbol{Vanilla}\ \mathit{Bond}$$

$$V_{i,t} = X_{i,t} \times D_{i,t} \times \left(\frac{1}{1 + \frac{Y_{i,t}}{200}} \right)^{\frac{\max(c_i - s, 0)}{c_i - c_i^-}} \times \frac{CPI(\mathrm{s})}{CPI(\mathrm{c_{i}, t})}$$

$$\mathit{If\ the\ bond\ is\ an}\ \boldsymbol{Inflation\ Linked}\ \mathit{Bond}$$

(12)

Where

$V_{i,t}$ is the value of $X_{i,t}$ on day *t*;

$Y_{i,t}$ is the applicable yield to maturity for bond *i* on day *t*;

c_i is the coupon payment date of bond *i* for its current Ex-Period;

c^-_i is the preceding coupon payment date;

s is the settlement date for *t*[13]

$D_{i,t}$ is the valuation date discount factor for bond *i* on day *t*.

---

[13] The max in the exponent is to prevent compounding (even if only for one or two days) of a coupon received

CPI($c_{i,t}$) is the CPI index ratio for the coupon payment date $c_{i,t}$.

CPI(s) is the CPI index ratio value for the settlement date *s*.

The total value of the Ex-Coupon Portion at any time is:

$$C_t = \sum V_{i,t}$$

(13)

4.4.11 Coupons being re-invested

The ex-coupon $X_{i,t}$ is due to be received on its payment date and will be re-invested at the first opportunity,

i.e. on the first trading day whose settlement date is on or after receipt date.[14] The coupon is assumed to be re-invested at the re-basing time, and at re-basing yields for the day. Until this happens, it is still part of the Ex-Coupon Portion; after the re-basing, it falls out of this portion.

The value of the ex-coupon for re-investment on the settlement date is, from the above,

$$\begin{array}{ll} R_{i,t} = V_{i,t} & \quad if\ t\ is\ the\ last\ day\ of\ the\ ExPeriod \\ R_{i,t} = 0 & \quad Otherwise \end{array}$$

(14)

[14] In other words, on the day defined above as being the last day of the Ex-Period. Note that, because of the lag between trade and settlement, the deals to re-invest the coupons will be done before the coupon is received.

4.4.12 Recalculation of the Nominal Factor, Kt

The recalculation of K_t takes place at the re-basing time. The new value, K'_t, holds thereafter.[15] Any changes in it reflect:

1. Changes in any of the weights. These could be due to re-weightings, re-constitutions or term sub-sector changes. Re-basings due to changes in weights take place on the last trading day before the weight change is effective;
2. The re-investment of coupons. These re-basings take place on the last day of the relevant ex-period.

K'_t is calculated such that the value of the portfolio based upon it (at closing prices) is the same as the closing value of the portfolio. This gives:

$$K'_t = \frac{B_t + \sum R_{i,t}}{\sum w'_{i,t} \times P_{i,t} \times D_{i,t}} \tag{15}$$

Where

K'_t is the new value of the factor;

B_t is the value of the Bond Portion at the re-basing yields for day t;

$R_{i,t}$ are the values of the coupons being re-invested on day t;

$w'_{i,t}$ are the new weights to obtain from the re-basing.

$P_{i,t}$ and $D_{i,t}$ are as defined above

[15] Generally, this would be from the next trading day. But, if real-time indices are to be calculated after the re-basing on the same day, the new value K'_t must be used. In this case too, the value of the coupons which have been re-invested must not be included in the total value, C_t, of the ex-coupons.

If we expand (15) we can see that K'_t is scaled so that the value of the bond portion of the portfolio plus cash coupons received on the day (the numerator) and the value of the bond portion of the portfolio after reweighting (the denominator) are the same. This ensures that value cannot be generated by re-weighting the portfolio when bond prices are kept constant. Obviously $R_{i,t} = 0$ after the reweighting because all coupons received are reinvested at the re-weighting.

$$1 = \frac{K_t \times \left(\sum w_{i,t} \times P_{i,t} \times D_{i,t}\right) + \sum R_{i,t}}{K'_t \times \left(\sum w'_{i,t} \times P_{i,t} \times D_{i,t}\right)}$$

(16)

4.4.13 Total Value of the Portfolio and the Index Value

The total value of the Portfolio at any time on day *t* is given by:

$$Z_t = B_t + C_t$$

(17)

where B_t and C_t are as defined above.

The value of the Index at the same time is:

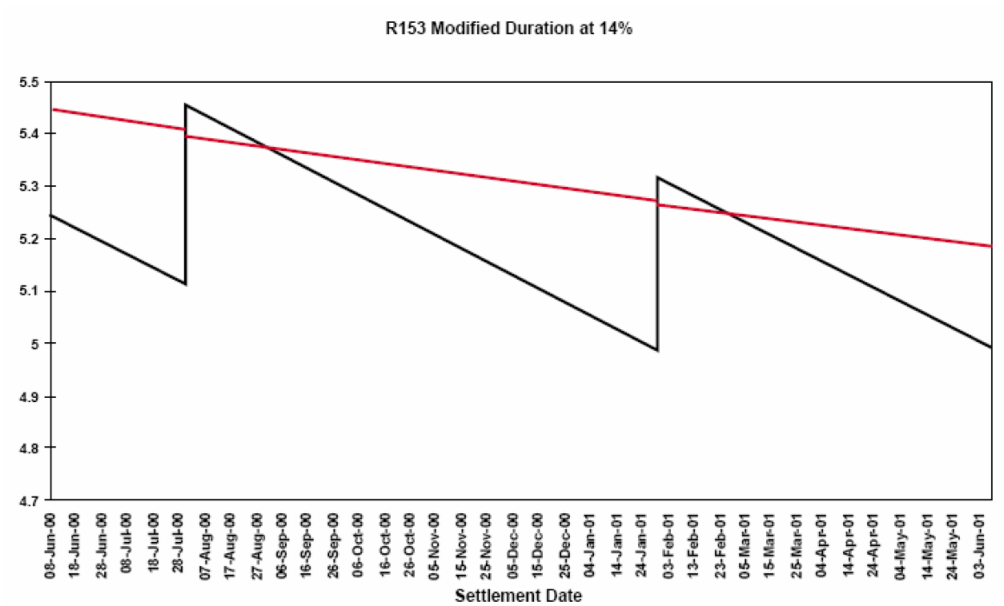
$$I_i = \frac{I_0}{Z_0} \times Z_t$$

(18)

4.5 Duration and Convexity

Duration and convexity require for their calculation the first and second differential of the bond or portfolio value with respect to yield to maturity. Given the somewhat complex and detailed definitions of this value given above, these differentials could be quite lengthy. However, whereas an exact valuation is crucial to the indices proper application, this is not the case with duration and convexity. These measures, which are usually quoted with only 2 or 3 significant digits, are more used to give an impressionistic view of a portfolio than to describe it precisely.

Furthermore, it is becoming widely accepted that these measures have some serious limitations for some applications. Both have the all-in price of a bond in the denominator of their formulae. As a bond goes ex-coupon, therefore, and its all-in price drops, so do duration and convexity show sudden increases. This is illustrated in the following graph, which shows how duration varies day by day over a period of a year, always at the same yield to maturity:



The zig-zag line shows the modified duration. There are some portfolios managers who feel that to be given a target for such an erratic measure is nonsensical. A possible solution is to use a "clean" duration, where the clean price replaces the all-in price in the denominator. This is the almost straight line in the diagram.

However, the clean duration itself has limitations. Historically, it was customary to use duration (and convexity) to estimate quickly a portfolio’s sensitivity to parallel shifts in the yield curve, via the application of Taylor’s Theorem to the total portfolio value. One cannot apply the clean measures in the same way: they must be applied to a clean value, which is not generally available.

Even though this use is now rare (with almost unlimited computer power it is simpler and more accurate to just re-value the portfolio), the appeal of the conventional measures lives on. Therefore, the duration and convexity measures of the indices will be the conventional, all-in ones.

But, in the light of the above, a couple of short-cuts will be taken in their calculation. The assumption will be made, in the calculation of duration and convexity according to the specifications of the Bond Pricing Formula, that bonds do not go ex-coupon, and hence are valued at all-in prices until their coupon payment day. The calculations will be made for the standard settlement day, with an adjustment to the valuation date. They will be performed daily on the portfolio after any re-basings for the day.

The virtue of these assumptions is that the portfolio can be treated as consisting only of its bond portion, and the ex-coupon portion can be safely ignored. Results will be exact as long as there are no re-basings between the real ex-coupon date and the coupon payment date; where there are re-basings, the percentage error will be negligible.

4.5.1 Modified Duration

The appropriate measure is the modified duration. Specifications can be found in the Bond Pricing Formula paper. As mentioned above, the calculation must assume that bonds do not go ex-coupon, and be for settlement on the standard settlement date.

The modified duration of the index or portfolio (they are the same) is given by:

$$dMod = \sum\left(N'_{i,t} \times P_{i,t} \times D_{i,t} + V'_{i,t}\right) \times \left(dMod_{i,t} + \frac{H_{i,t}}{2\left(1 + Y_{i,t}/200\right)}\right)/Z_t \tag{19}$$

where $dMod_{i,t}$ is the modified duration of bond i for day s, Z_t is the total portfolio value at the time, from Equation 17 above $N'_{i,t}$ is the nominal amount of bond i in the portfolio, effective on or before the next trading day (ie after any re-basing on day t), $V'_{i,t}$ are the values of the ex-coupon portions from Section 4.4.10, after deducting any re-investments for the day (i.e. $V'_{i,t} = V_{i,t} - R_{i,t}$). $P_{i,t}$, $D_{i,t}$, $H_{i,t}$ and $Y_{i,t}$ are all as defined above.

4.5.2 Convexity

The convexity of the portfolio or index follows similarly:

$$Conv = \sum\left(N'_{i,t} \times P_{i,t} \times D_{i,t} + V'_{i,t}\right) \times \left(Conv_{i,t} + \frac{H_{i,t} \times dMod_{i,t}}{\left(1 + Y_{i,t}/200\right)} + \frac{H_{i,t}\left(2H_{i,t} + 1\right)}{4\left(1 + Y_{i,t}/200\right)^2}\right)/Z_t \tag{20}$$

Where $Conv_{i,t}$ is the convexity of bond *i* calculated for *t*'s settlement date, s, assuming it does not go ex; and the remaining variables are as defined above

4.5.3 Use

As discussed above, these measures can be used to estimate the change in value of the index for a given parallel shift in the yield curve. From Taylor's Theorem, we get:

$$\Delta I = I \times (-\Delta y \times dMod + \frac{1}{2}\Delta y^2 \times Conv) \tag{21}$$

Where

ΔI is the estimated change in the index

I is the current index value

Δy is the shift in yield, expressed here as a decimal (100 basis points is .01)

4.6 Precision and Rounding

All values are calculated and held to IEEE double precision (15 significant digits).

All-in and clean prices are rounded to 5 decimal places in accordance with the Bond Pricing Formula specifications.

Index values will be published rounded to three decimal places. This will allow calculation of returns to 2 decimal places (when expressed as percentages). A one basis point change in all yields to maturity will cause a change of about .05 in the index; the same change applied to the lowest market capitalisation constituent will not be observable in the third decimal place.

Durations will be published to two decimal places, and convexities to one.

APPENDIX A: Clean Price Index

The price index is a representation of the capital value of the bond portfolio without regard for the interest accrued or coupons paid. This index is "clean" and is calculated as follows:

$$I_t = \frac{k \sum_{i=1}^{n} P_{i,t} \times W_{i,t}}{\sum_{i=1}^{n} W_{i,t}}$$

Where

I_t is the value of the index at time t;

$P_{i,t}$ is the clean price per R100 nominal value of constituent i at time t;

n is the number of constituents in the index;

k is a constant chosen to base the index at 100 as at the starting date of the index. i changes when constituents or weightings change so that the value of the index is unaffected by the change at the instant of the change.

Thus the new constant, k', is calculated such that: $I'_t = I_t$ where I'_t represents the index after the change.

APPENDIX B: All in Price Index

The all in price index is identical to the clean price index save for the fact that the all in price is used. This should be similar to the performance of a bond portfolio that does not reinvest its coupons but rather pays them out to the investor.

$$I_t = \frac{k \sum_{i=1}^{n} P_{i,t} \times W_{i,t}}{\sum_{i=1}^{n} W_{i,t}}$$

Where

I_t is the value of the index at time t;

$P_{i,t}$ is the all in price per R100 nominal value of constituent i at time t;

n is the number of constituents in the index;

k is a constant chosen to base the index at 100 as at the starting date of the index. i changes when constituents or weightings change so that the value of the index is unaffected by the change at the instant of the change.

Thus the new constant, k', is calculated such that: $I'_t = I_t$ where I'_t represents the index after the change.

APPENDIX C: Yield Indices

Interest Yield Index

The interest yield index is produced for legacy reasons and provides a mean yield estimate taken directly from the publication *The JSE Actuaries Bond Indices* which was introduced in 1988.

The Interest Yield on day *t* is defined as the annual interest yield on the price index, i.e. interest receivable over a full year divided by the market capitalisation. It is calculated as:

$$G_t = \frac{\sum_{i=1}^{n} G_{i,t} \times W_{i,t}}{\sum_{i=1}^{n} P_{i,t} \times W_{i,t}}$$

Where G_t is the Interest Yield at time *t*; and is the coupon per R100 nominal value of bond *i* at time *t*.

Interest yield is not calculated for inflation linked bonds as the CPI inflated nominal amount of the bond makes interpretation of the interest yield nonsensical.

Inflation Linked Interest Yield Index

The inflation linked interest yield index functions as an approximate yield measure in the inflation linked space. It gives an indication of the average annual real yield earned by the portfolio and is calculated as follows:

$$Y_t = \frac{\sum_{i=1}^{n} Y_{i,t} \times P_{i,t} \times W_{i,t}}{\sum_{i=1}^{n} P_{i,t} \times W_{i,t}}$$

Where

Y_t is the weighted average yield of the index at time *t*

$Y_{i,t}$ is the real yield of the i^{th} bond at time *t*

$P_{i,t}$ is the all in price of the i^{th} bond at time *t*

$W_{i,t}$ is the nominal weight of the i^{th} bond in the index at time *t*

Average Yield Index

The average yield index calculates the one yield that when used to value every bond in the index, produces the same index level. It is therefore, the average yield attained by the index. For the vanilla indices, the average yield is calculated using only 5 iterations the Newton-Bailey method as illustrated below. As a note, only positive iterations of the yield are considered. Negative values are set to zero:

$$Y_{i+1} = \left(\frac{Y_i}{100}\right) - \frac{I - I_i}{(I_i \times dMod_i) - \frac{\left((I - I_i) \times Conv_i\right)}{(2 \times dMod_i)}}$$

Where

Y_i is the i^{th} iteration of the average yield (Y_0 is the largest of the constituent bond yields)

I_i is the i^{th} iteration of the portfolio value calculated using Y_i as the yield for all of the constituent bonds

I is the original index value which remains a constant

$dMod_i$ is the modified duration of the index I_i

$Conv_i$ is the convexity of the index I_i

5 Credits

The compilation of the existing Fixed Income Indices were managed by Dr James Greener and built in conjunction with the Actuarial Society of South Africa (ASSA), leading up to the initial index launch in July 2000. The indices and the original document from which the majority of this document draws its content were originally defined by Rogan Etheridge of Quant Financial Research. Please refer to BEASSA Fixed Income Indices, August 2000, ASSA and BESA. Professor Rob Thomson of the University of Witwatersrand provided the original impetus and mathematical principles via the Investment Committee of the Actuarial Society of South Africa.

Several gatherings of market participants were held on the course of preparing the original specification and their contributions are also noted with thanks. In particular, Jill Mann of Rand Merchant Bank acted as a continual spur to various participants in the process to keep focus and momentum. Glen Manning of JP Morgan provided advice and valuable information on the JP Morgan bond Indices.

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