

ALWAYS. DERIVATIVE RESEARCH

THE IMPLIED VOLATILITY INDEX FUTURE
WITH APPLICATIONS

THE SAFEX INTERBANK VOLATILITY INDEX FUTURE
(SIVXF)

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EXECUTIVE SUMMARY

This report is a continuation of our previous work on the construction and applications of an implied volatility index, the SIVX (Safex Interbank Volatility Index), to the South African financial markets.

With this volatility index platform in place, the next step is to consider the introduction of an exchange traded volatility index futures' contract and test its' performance in an asset management setting. The research that follows addresses this issue.

A volatility index future is a unique financial instrument in that it allows the participant to trade implied volatility directly.

For example: An option's price is (amongst other things), affected by the underlying asset price, implied volatility, the option strike and time. An implied volatility futures' price, however, depends only on time and the relevant level of implied volatility.

The study shows that the proposed volatility futures contract can be used as an efficient instrument to:

- Reduce the level of kurtosis in a portfolio's return distribution (and in general mitigate the risks implicit in the higher moments).
- Provide an alternative hedging instrument to Put options.
- Indicate the relative value in options given the term structure of volatility.

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1. MOTIVATION FOR THE RESEARCH

Subsequent to the introduction of the SAFEX Interbank Volatility Index (SIVX) in our November 06 report, this research investigates the construction and use of an implied volatility index futures contract.

Volatility futures provide exposure to an “implied volatility” asset class. Authors such as Bowler (2004) and Black (2004) discuss the merits of such an alternative investment in a fund management setting. The analysis that follows, applies their work to the South African financial markets.

2. A NOTE ON VARIANCE AND VOLATILITY

In his book on “Asset Pricing Dynamics”, Taylor mentions that the variance of a multi-period return is the *sum of single period variances* when the random walk hypothesis is true. His definition of the random walk hypothesis incorporates “the idea that prices wander in an unpredictable manner” over time. Shalen and Hiatt (2004) adapt this theory to arrive at a formula for a forward variance measure which can be transformed into a forward volatility measure by taking it’s square root.

The relationship between forward variance and spot variance is similar to that of a forward interest rate and a spot interest rate. Derman (1998) expands on this comparison using volatility. He uses the term “local volatility” to refer to a forward volatility measure that occupies part of the overall volatility term structure, in the same way as an interest rate future occupies a “local” part of the zero curve.

3. THE VOLATILITY FUTURE - INTRODUCTION

Volatility futures contracts enable a direct investment in implied volatility - as opposed to a variance swap which offers a direct investment in historical or realised volatility, Derman (1998).

Let’s consider the salient features of a **near dated** volatility futures contract:

CONTRACT NAME:

SAFEX Inter-Bank Volatility Index Future (SIVXF)

CONTRACT EXPIRY:

The same day and time as the existing quarterly SAFEX futures expiries: March, June, September and December.

RANGE OF CONTRACTS:

The SIVXF is a proposed near dated futures contract on the SIVX implied volatility index. Pricing a volatility future beyond that of a near contract would require sufficient longer dated liquidity in the underlying TOP40 option market.

CONTRACT SIZE:

- A SIVXF value of 21.32 has an equivalent percentage representation of 21.32% (0.2132) or a basis point representation of 2132 (0.2132 x 10,000).
- The SIVXF has a Rand value given that **1 basis point represents R10**. Hence a SIVXF value of 21.32, for example, will have a Rand value of R21,320 (0.2132x 10,000x R10).

DELIVERY:

- The SIVXF is a cash-settled contract and would be margined by the exchange on a daily basis. On expiry, the SIVXF and the SIVX converge, given the mathematical relationship between the two. The final margin cash balance on expiry is therefore a function of the original SIVXF level transacted at and the SIVXF expiry value, multiplied by the number of contracts and the multiplier of R10.

For example: Assume 15 SIVXF contracts are bought 1 month prior to expiry at an assumed SIVXF level of: 22.15. Next, assume the SIVXF on expiry is 22.50. The net surplus in the margin account on expiry, for the purchaser of this contract, should be: $(-0.2215+0.2250) \times (10,000 \times R10) \times 15 = R5,250$.

MARGIN REQUIREMENTS:

- The initial margin requirement will be "x %" of nominal traded as stipulated by the exchange. The nominal value of the 15 contract position mentioned in the preceding example is: $0.2215 \times (10,000 \times 10) \times 15 = R332,250$. If the initial margin is set at 10%, say, then the amount to lodge with the exchange will be: $0.1 \times R332,250 = R33,225$ or R2,215 per contract.
- The change in value of a volatility futures position from one day to the next would typically equate to the variation margin required on a daily basis by the exchange.

4. VOLATILITY INDEX FUTURES CONSTRUCTION

This report references the method adopted by Shalen and Hiatt (2004) for the construction of the SIVXF. The data required as input to the forward volatility model is as follows:

- Daily implied variances for the near contract (based on near at the money implied volatility)
- Daily implied variances for the next near contract (based on next-near at the money implied volatility)

Shalen and Hiatt (2004) make the point that raw implied volatility data needs to be de-annualised before it is used to calculate the implied variance. Implied volatility is typically in annualised form, when implied from the price of an option. Squaring the de-annualised implied volatility provides the required implied variance time series data.

These variances are then used in the formula detailed in Appendix 2 to calculate the forward implied variance. Forward implied volatility is obtained by taking the square root of the forward variance. (Note: With regard to the formula in Appendix 2, the de-annualisation factor of 252 is not shown as it cancels out once the time weights are applied).

From a technical point of view, Shalen and Hiatt (2004) mention that the implied forward volatility calculated using the square root of de-annualised variances "is not an exact fair value for the volatility future, it is only an upper bound". They illustrate, however, that the forward volatilities calculated in this way can provide a good "approximation to the fair value of the volatility future".

This research uses the “upper bound” approach as a sufficient estimate of the fair value of the SIVX future (SIVXF). A more technical discussion on this may be found in Dupire (2004).

The time weightings used in the construction of the SIVXF are approximations relating to actual work days not calendar days. For the interested reader, Fleming (1995) has more to say on this.

Table 1 illustrates the daily price evolution of the SIVX (spot) and SIVXF (near future) for the third quarter of 2006.

TABLE 1: DAILY SIVX VS. SIVXF

Quarter Three - 2006				
Date	Near ATM Vol.	Next Near ATM Vol.	SIVX	SIVXF
19/06/2006	25.52%	23.90%	25.60%	22.08%
20/06/2006	27.90%	25.53%	27.97%	22.84%
21/06/2006	28.46%	25.92%	28.50%	23.05%
22/06/2006	28.01%	25.61%	28.01%	22.96%
26/06/2006	27.64%	25.36%	27.57%	22.93%
27/06/2006	27.86%	25.51%	27.76%	23.04%
28/06/2006	28.10%	25.67%	27.95%	23.16%
29/06/2006	28.26%	25.78%	28.07%	23.26%
30/06/2006	28.13%	25.69%	27.90%	23.26%
03/07/2006	26.47%	24.55%	26.26%	22.71%
04/07/2006	26.57%	24.62%	26.33%	22.78%
05/07/2006	26.66%	24.68%	26.38%	22.85%
06/07/2006	26.62%	24.66%	26.32%	22.87%
07/07/2006	26.53%	24.60%	26.20%	22.86%
10/07/2006	26.41%	24.52%	26.06%	22.85%
11/07/2006	26.61%	24.65%	26.22%	22.97%
12/07/2006	26.93%	24.87%	26.49%	23.13%
13/07/2006	27.08%	24.97%	26.59%	23.23%
14/07/2006	27.02%	24.93%	26.51%	23.24%
17/07/2006	26.64%	24.67%	26.12%	23.11%
18/07/2006	27.03%	24.94%	26.45%	23.31%
19/07/2006	28.18%	25.72%	27.46%	23.83%
20/07/2006	29.61%	26.70%	28.71%	24.49%
21/07/2006	30.63%	27.39%	29.58%	24.96%
24/07/2006	30.44%	27.26%	29.36%	24.94%
25/07/2006	30.84%	27.53%	29.67%	25.17%
26/07/2006	30.73%	27.46%	29.52%	25.18%
27/07/2006	30.79%	27.50%	29.53%	25.26%
28/07/2006	31.12%	27.72%	29.76%	25.47%
31/07/2006	30.45%	27.27%	29.13%	25.22%
04/08/2006	26.63%	24.66%	25.69%	23.57%
07/08/2006	26.81%	24.79%	25.82%	23.70%

TABLE 1: DAILY SIVX VS. SIVXF (continued)

Quarter Three - 2006				
Date	Near ATM Vol.	Next Near ATM Vol.	SIVX	SIVXF
08/08/2006	26.50%	24.58%	25.52%	23.57%
10/08/2006	26.22%	24.38%	25.23%	23.49%
11/08/2006	26.31%	24.44%	25.28%	23.56%
14/08/2006	26.50%	24.58%	25.40%	23.70%
15/08/2006	26.08%	24.29%	25.03%	23.50%
16/08/2006	25.61%	23.96%	24.62%	23.27%
17/08/2006	25.36%	23.79%	24.39%	23.15%
18/08/2006	25.31%	23.75%	24.33%	23.15%
21/08/2006	25.50%	23.88%	24.46%	23.29%
22/08/2006	25.32%	23.76%	24.29%	23.21%
23/08/2006	24.82%	23.42%	23.87%	22.95%
24/08/2006	24.74%	23.36%	23.79%	22.92%
25/08/2006	24.92%	23.48%	23.90%	23.05%
28/08/2006	24.82%	23.42%	23.81%	23.01%
29/08/2006	24.74%	23.36%	23.72%	22.99%
30/08/2006	24.74%	23.36%	23.70%	23.01%
31/08/2006	24.83%	23.42%	23.75%	23.09%
01/09/2006	24.97%	23.52%	23.83%	23.20%
04/09/2006	25.57%	23.31%	23.77%	22.84%
05/09/2006	25.75%	23.32%	23.77%	22.84%
06/09/2006	25.89%	23.35%	23.78%	22.89%
07/09/2006	26.00%	23.42%	23.82%	23.00%
08/09/2006	26.07%	23.54%	23.89%	23.17%
11/09/2006	26.12%	23.72%	24.02%	23.41%
12/09/2006	26.08%	23.57%	23.84%	23.28%
13/09/2006	26.01%	23.37%	23.61%	23.11%
14/09/2006	25.92%	23.21%	23.42%	22.99%
15/09/2006	25.81%	23.21%	23.37%	23.04%
18/09/2006	25.34%	23.31%	23.40%	23.21%
19/09/2006	25.15%	23.22%	23.27%	23.15%
20/09/2006	24.94%	25.47%	25.46%	25.48%
21/09/2006	24.71%	27.73%	27.73%	27.73%

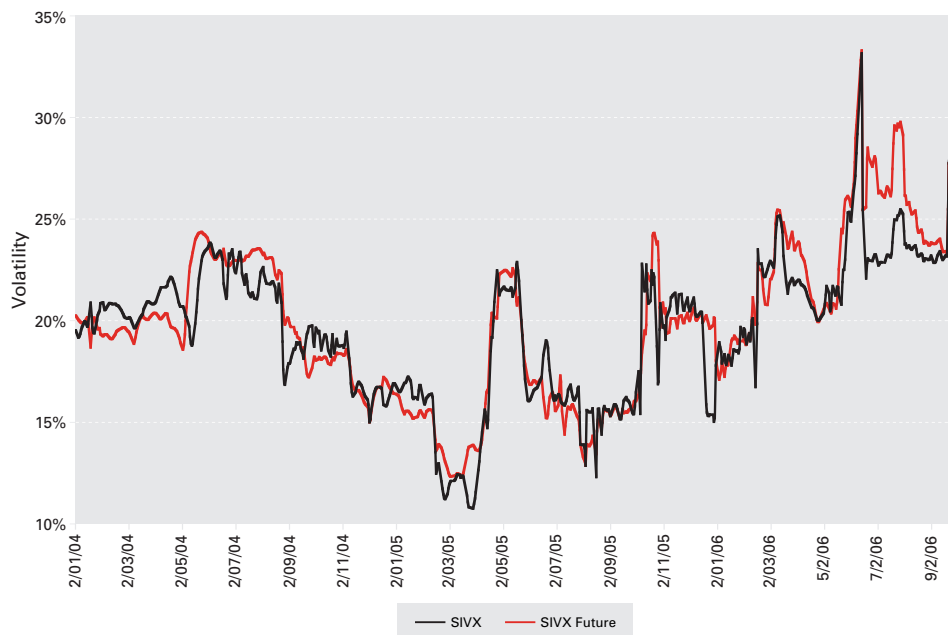
In Table 1, the SIVXF represents a daily forecast/expectation of three month implied volatility (SIVX) on the expiry date of the near futures contract.

Notice the convergence of the volatility future with spot (SIVX) on expiry.

5. THE SIVXF: SAFEX INTERBANK VOLATILITY INDEX FUTURE

Figure 1 plots the SIVXF and the SIVX on a daily basis from Jan 2004 to Sep 2006.

FIGURE 1: SIVX VS. SIVXF (NEAR FUTURE)



Notice in Figure 1 the premiums and discounts at which the SIVXF trades relative to spot (SIVX), during any one quarter.

Bowler (2004) explains this: “Implied volatility products are designed to settle into implied volatility upon expiry of the contract. However, prior to this, its value should reflect the market’s expectation of what implied volatility will be on expiry of the contract. This is termed forward implied volatility. In a low volatility environment an implied volatility contract will tend to price higher than the spot volatility and will converge to spot from above”. The opposite should be true in a high volatility environment.

Shalen and Hiatt (2004) comment on the premiums and discounts as follows: “Whether the futures price is at a premium or discount relative to the underlying implied volatility index depends on whether or not the market believes volatility will be higher or lower in the near three month period”.

The discounts and premiums have a term structure implication too. Bowler (2004) mentions that the term structure is typically upward (downward) sloping when short dated volatility (SIVX) is below (above) its long term expectation (SIVXF). Monitoring this can show “overvalue” or “undervalue” in puts relative to a volatility futures hedge. This is discussed in more detail in the applications that follow.

6. SIVXF APPLICATIONS

PORTFOLIO HEDGE

This section considers the approach used by Bowler (2004) to construct a “hedged” portfolio with exposures to both the S&P and the VIX, starting in 1986. The same methodology is then applied to a TOP40 portfolio using our newly constructed SIVXF contract.

Bowler’s results show that “a hypothetical investment consisting of a 10% long position in the spot VIX index combined with a 90% allocation to the S&P 500 index, rebalanced weekly, would have outperformed the S&P500 by approximately 5% per year with 25% lower risk since 1986.”

Black, K (2004) produces similar results with an 85% S&P, 15% VIX combination from Jan 1994 to April 2004. In this study, particular attention is drawn to the impact that volatility futures can have on the higher moments of a portfolio’s return distribution.

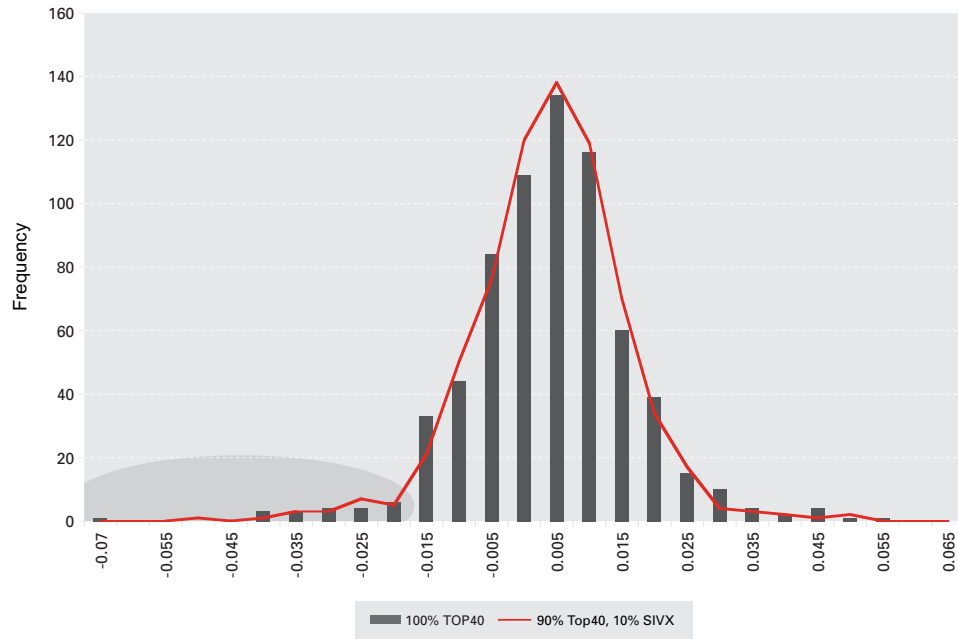
Following their example, a portfolio is constructed with fixed percentage investments in the TOP40 and SIVX futures, rebalanced weekly, from Jan 2004 to Sep 2006. The initial portfolio value in Jan 2004 is R100,000,000. Table 2 details the results.

TABLE 2: PERFORMANCE OF COMBINED TOP40 & SIVXF PORTFOLIO, WEEKLY RE-BALANCE

TOP40	SIVX	Skewness	Kurtosis	Standard Dev	Portfolio Value Sep 06
100%	0%	-0.1354	5.57	20.00%	255,014,192
95%	5%	-0.1754	5.51	18.87%	258,619,437
90%	10%	-0.1855	5.29	18.32%	261,414,005
85%	15%	-0.1320	5.10	18.38%	263,371,850
80%	20%	-0.0081	5.54	19.05%	264,474,166

From this table and the results of the aforementioned authors, there is sufficient evidence to suggest that the introduction of volatility futures into the portfolio mix can provide downside protection and mitigate the impact of higher moments on a portfolio’s return distribution. Figures 2 and 3 illustrate this graphically.

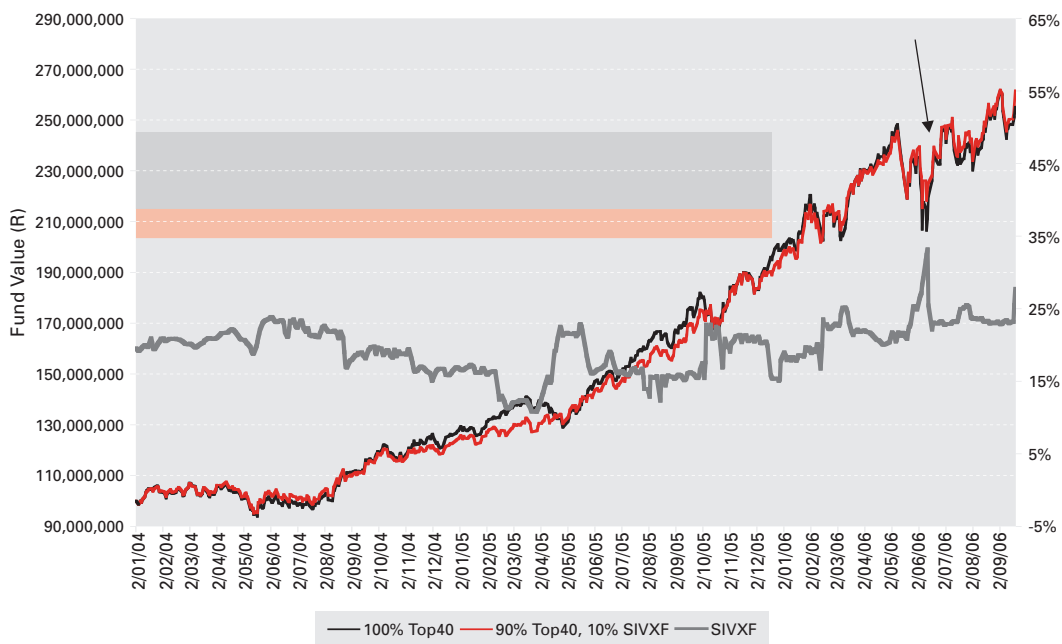
FIGURE 2: 100 PORTFOLIO VS. 90/10 PORTFOLIO (TOP40/SIVXF) RETURN DISTRIBUTION



In Figure 2, the left ellipse illustrates the “fat” left hand side tail of the histogram for the 100% TOP40 portfolio i.e. higher kurtosis. The kurtosis is reduced with the introduction of volatility futures in the 90% TOP40/10% SIVXF portfolio (red line).

Figure 3 plots the daily value of the 90/10 portfolio (red line) against that of the 100 portfolio (black line) from January 04 to September 06.

FIGURE 3: 100 PORTFOLIO VS. 90/10 PORTFOLIO (TOP40/SIVXF)



The left hand axis in Figure 3 refers to the daily Rand value of the 100% TOP40 futures portfolio and the 90% TOP40 futures, 10% SIVXF combination. The right hand axis illustrates the daily value of the volatility future (SIVXF), plotted in grey.

The grey shaded area represents the value lost by the 90/10 portfolio when the markets dropped suddenly at the end of the second quarter of 2006, while the red shaded area illustrates the value lost by the 100 portfolio in addition to the loss incurred by the 90/10 portfolio over the same period.

The question may be asked as to why the long volatility futures position did not start protecting the 90/100 portfolio sooner. Bowler (2004) helps to explain: “Forward volatility becomes less sensitive to changes in spot-implied volatility the further forward it becomes. Hence the futures will not rise as fast as the index when markets decline significantly, if they have a long time to mature.” i.e. If the SIVXF was a future on a one month implied volatility expectation (and not three month), its price changes would respond quicker to price changes in the SIVX. A further contributing factor to this speed of adjustment, says Bowler, is the fact that forward volatility is particularly sensitive to the slope of the volatility term structure at any one point in time.

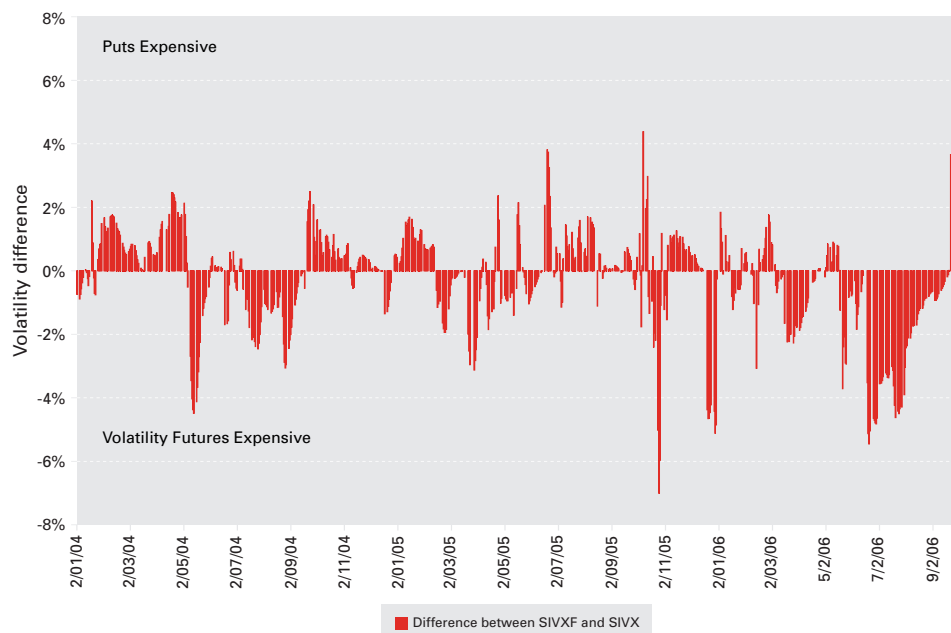
RELATIVE VALUE INDICATOR

Black, K (2004) outlines the benefits of using volatility futures relative to put options for downside protection. He claims that the cost of a volatility futures hedge relative to an ATM Put hedge depends on the term structure of implied volatility. In particular:

- Futures have a higher cost when forward volatility is greater than today's volatility (i.e. $SIVXF > SIVX$) and
- Futures have a lower cost when the forward volatility is less than today's volatility (i.e. $SIVXF < SIVX$).

Figure 4 plots the *difference* between SIVX and SIVXF ($SIVX - SIVXF$). The positive area suggests that puts are being priced off volatility higher than that implied by the future. Alternatively, for negative differences Puts are being priced off lower volatility relative to the current volatility forward i.e. volatility futures are relatively more expensive.

FIGURE 4: DIFFERENCE BETWEEN SIVX AND SIVXF ($SIVX - SIVXF$) JAN 04 - SEP 06



7. FURTHER RESEARCH TOPICS

Two interesting applications of volatility futures not discussed in detail in this report are:

Portfolio Insurance:

- In 1988 Perold and Sharpe published a classic paper on Dynamic Strategies for Asset Allocation. In this paper they discussed an asset manager's exposure to portfolio insurance depending on the way in which a portfolio is re-balanced. For example:

Managers that tend to sell into rising markets and buy into falling markets “give rise to **concave** payoff curves (i.e. these curves increase at a decreasing rate as one moves from left to right). They don’t have much downside protection and do relatively poorly in up markets. However, these managers generally do very well in flat but oscillating markets”. Their profile is that of a “seller of portfolio insurance”. Similarly, asset managers that sell into falling markets ie “go longer into cash” are classified by Perold as buyers of portfolio insurance.

Volatility futures can be used to mitigate the impact of volatile or non-volatile markets on specific portfolio insurance strategies.

Volatility Swaps:

- When implementing a volatility swap the fixed leg is typically a function of the implied volatility term structure at the start date concerned. Assuming a manager pays fixed (strike) and receives floating on a volatility swap that matures at the end of each quarter. Under this structure the manager is exposed to implied volatility increasing. As a result, it is possible that a higher fixed level on the swap is payable, relative to the previous quarter.

The risk of locking in progressively higher fixed swap levels, as implied volatility increases over time, can be mitigated at the start of each quarter. This is done by paying the fixed leg of the swap and simultaneously buying the three month volatility future. Hence, in this example, a short dated volatility swap combined with a long volatility future’s position, stabilises the fixed leg of the swap.

A constant longer term volatility exposure is thus achieved for a chosen portfolio of assets.

8. CONCLUSION

In this report we discuss the construction and applications of the SAFEX Interbank Volatility Index Future (SIVXF). This futures contract is a daily forecast of where the “spot” volatility index (SIVX) will be trading on the near SAFEX expiry date.

Two applications of the volatility futures contract are discussed within a South African “emerging market” setting. These are:

- A Portfolio Hedge – The results are referenced against independent research by Black (2004) and Bowler (2004). The research shows that SIVX futures can mitigate higher moment risk as well as hedge against falling markets, for a given TOP40 portfolio.
- An Alternative Value Indicator –Volatility futures can provide an alternative hedging instrument to put options based on their relative value (for a given implied volatility term structure).

Further areas for research relating to “forward volatility” are briefly discussed, with particular reference to Portfolio Insurance and Volatility Swaps.

From a market making perspective, Appendix 3 contains a simplified numerical analysis of a volatility futures hedge. This is a technical topic that is discussed in more detail in Kani (1996) and Derman (1998).

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APPENDIX 1 - CALCULATING THE SIVX

NOTATION

- T_1 = Near Top40 Futures expiry
- $\sigma(0, T_1)$ = Implied ATM volatility of options on Near Top40 Futures contract
- T_2 = Next Near Top40 Futures expiry
- $\sigma(0, T_2)$ = Implied ATM volatility of options on Next Near Top40 Futures contract

METHOD 1

Whaley (2000) calculates the SIVX by interpolating between the Near and Next-Near ATM implied volatilities. This approach is used to create a three month implied volatility as follows:

$$SIVX_{VOL} = \sigma(0, T_1) \left[\frac{N_{T_2}}{N_{T_2} - N_{T_1}} - 1 \right] + \sigma(0, T_2) \left[1 - \frac{N_{T_1}}{N_{T_2} - N_{T_1}} \right]$$

where N_T = trading days till T .

METHOD 2

Alternatively, consider the additive properties of variances: this involves interpolating between the Near and Next-Near ATM implied variances to create a three month implied variance. Taking the square root gives the required three month implied volatility:

$$SIVX_{VAR} = \sqrt{\sigma^2(0, T_1) \left[\frac{N_{T_2}}{N_{T_2} - N_{T_1}} - 1 \right] + \sigma^2(0, T_2) \left[1 - \frac{N_{T_1}}{N_{T_2} - N_{T_1}} \right]}$$

where N_T = trading days till T .

Figure 5 compares the two approaches graphically.

FIGURE 5: "VOLATILITY" SIVX VS "VARIANCE" SIVX



In Figure 5 the solid line represents the SIVX as calculated by Method 1, ie. Whaley’s formula (uses implied volatilities) and the dots represent the corresponding SIVX points using Method 2 (implied variances).

The graph illustrates that the two methods track each other almost identically. For consistency, this report uses Method 1.

APPENDIX 2 - CALCULATING THE SIVXF

As mentioned earlier in this report, Taylor (2005) states that the variance of a multi-period return is the sum of single period variances when the random walk hypothesis is true. Using this theory and applying a weighting scheme we are able to isolate the forward volatility, given near and next-near volatility as follows:

$$SIVXF = \sqrt{\sigma^2(0, T_2) \left[\frac{N_{T_2}}{N_{T_2} - N_{T_1}} \right] - \sigma^2(0, T_1) \left[\frac{N_{T_1}}{N_{T_2} - N_{T_1}} \right]} \quad (\text{see notation Appendix 1})$$

where N_T = trading days till T .

Shalen and Hiatt (2004) state that the implied forward volatility calculated using the square root of de-annualised variances "is not an exact fair value for the volatility future, it is only an upper bound".

APPENDIX 3 - ANALYSIS OF A SIVXF HEDGE

Section 2 of this report outlines Derman's description of the forward volatility measure as "local volatility". The theory of local volatility and the hedging thereof is discussed in more detail in his paper with Kani (1996): "The Trading and Hedging of Local Volatility". Their work in this area is quite technical, hence the discussion that follows applies a simpler version of the original paper, presented in Derman (1998).

Derman (1998) introduces the concept of a "volatility gadget". These gadgets contain specific options that are combined to hedge local volatility risk, in each future time period, for each index level, all at zero cost. In particular, he illustrates the application of "a long calendar spread and a short butterfly spread" in the underlying equity market index, to hedge the local volatility concerned.

- For example: calendar and butterfly spreads would be held in S&P index options to hedge VIX futures positions.

In Table 3 Derman's gadget structure is applied to the hedging of a long volatility futures contract on the SIVXF, held for a full quarter. Several quarters are chosen to empirically test the hedge results.

The example assumes that the volatility trader is long 10 SIVXF contracts at the start of the quarter concerned. TOP40 calendar and butterfly spreads (i.e. the gadgets) are used to hedge this exposure on a zero premium basis. A simplifying assumption is that one butterfly and one calendar spread "structure" is implemented per expiry.

Table 3 summarises the hedge results for five quarters (non-consecutive). The optimal number of gadgets to trade in each quarter is calculated using a genetic algorithm. The algorithm tests for the number of gadgets that keep the profit and loss on the combined futures and hedge (gadget) position (at expiry) as close to zero as possible. In this way the trader gets a sense of the number of gadgets that would have resulted in a reasonable hedge trade, for the quarter concerned.

This is a naïve initial attempt at analysing a SIVXF hedge. A second level of investigation would be to consider the hedge outcome based on vega neutralising the combined exposures at the start of each quarter. This is not covered here.

TABLE 3: QUARTERLY CASHFLOWS FROM A "GADGET" HEDGE

Quarter Trade Date	18/06/2004	17/09/2004	17/12/2004	17/06/2005	17/03/2006
Initial SIVX	21.82%	18.12%	15.86%	18.28%	23.17%
End SIVX	18.44%	16.65%	12.36%	15.64%	25.45%
Initial Near Futures	9,380	10,683	11,281	13,068	18,030
End Near Futures	10,568	11,140	12,595	14,745	18,000
Initial Next Near Futures	9,475	10,774	11,335	13,155	18,160
End Next Near Futures	10,640	11,253	12,676	14,826	18,215
Butterfly Spread LHS Strike	8,800	10,150	10,750	12,450	17,000
Butterfly Spread ATM Strike	9,400	10,700	11,300	13,100	18,000
Butterfly Spread RHS Strike	10,000	11,250	11,850	13,750	19,000
Number of gadgets (# Butterflies = # Calender Spreads)	64	14	33	42	31
Butterfly Spread Premium	R 102,770	R 21,482	R 52,132	R 85,563	R 88,909
Calender Spread Premium	-R 104,103	-R 22,292	-R 55,023	-R 89,471	-R 100,857
Net Option Spreads Premium	-R 1,333	-R 809	-R 2,891	-R 3,907	-R 11,948
# SIVX Futures	10	10	10	10	10
Settlement Date	16/09/2004	15/12/2004	17/03/2005	15/09/2005	15/06/2006
SIVXF P&L	-R 33,876	-R 14,656	-R 35,040	-R 26,417	R 22,742
Butterfly Spread P&L	R 102,770	R 6,082	R 52,132	R 85,563	-R 221,091
Calender Spread P&L	-R 68,632	R 9,134	-R 16,535	-R 58,872	R 198,713
Net Cumulative P&L (Quarter end)	R 261	R 561	R 558	R 275	R 363

In Table 3 the number of gadgets are optimised by the genetic algorithm.

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