Building better markets

BOND PRICING FORMULA

Specifications

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INTRODUCTION

South African bonds are quoted and traded in yield[1], but, of course, are settled in price. This means that there must be a standard convention for converting between the yield and the price of a bond for a given settlement date.

The presently accepted way of doing this is the Bond-Pricing Formula of the JSE's Gilt Clearing House (“the GCH formula"), introduced in 1984. The GCH formula (and subsidiary conventions which have grown up around it) is the subject of this paper.

Note that this paper does no more than describe present market practice. Programmers do not have to re-program their code! Programmers programming anew can use these specifications, and will get the same results as anyone else in the market.

There have been occasional moves to change the formula, which does suffer from a number of (rather small) anomalies. The chief of these are: the formula is not consistent with the pricing of NCDs; it does not give a clean price of 100 for a bond trading at par between coupon dates; its yield is conceptually different to that used by accountants in amortising an investment in a bond; and the rounding conventions appear somewhat arbitrary. Programmers should, therefore, be aware of the possibility of change in future, and modularise / parameterise their code accordingly. They should also contact the Bond Exchange before commencing, to ensure that they have the latest version of the specifications.

When speaking of the price or yield of a bond, one should always stipulate the settlement date. A yield of a bond is a yield "for settlement on such-and-such a settlement date", and is converted to a price "for settlement on [the same] settlement date." The treatment below, however, assumes that the settlement date is given, and therefore applies whatever settlement date convention is used.

The GCH formula only applies to "conventional" bonds - ie. those paying fixed (including zero) coupons, and with two coupon payment dates per year, one of which coincides with an anniversary of the bond's maturity date, on which date the whole capital of the bond is redeemed. Bonds which fail to meet one or more of these criteria cannot be valued using the GCH formula.[2] Some of these bonds are priced using various other formulae. Most of them are "price stocks", and are traded on the basis of price rather than yield. In order to accommodate price stocks, the Bond Exchange’s capture system allows trades to be input with either a yield, or an all-in consideration.[3]

The Appendix provides a "fast track" for those with some knowledge of the markets. It expresses the core GCH formula as succinctly as possible. The main text is far more discursive and descriptive, and includes results and methods which are not part of the core formula. It will be useful to those with less experience of the markets, and to programmers writing general-purpose systems. Results which are in common to the two approaches will be identical.

[1] The precise term is “yield-to-maturity”, indicating that the yield encompasses all future cash flows on the bond. We shall use the two terms interchangeably.

[2] A partial exception regarding multiple maturity dates is discussed in footnote 7 on page 2.

[3] Or both, in which case the consideration takes precedence.

1 INPUTS

It is required to find various price information for a bond, at a given yield-to-maturity on a settlement date:

- Yield-to-maturity [4] — Y
- Settlement date [5] — S
- Information for the bond, $B = BOND(M_B, C_B, R_B, CpnDates, BcDates)$: [6]
  - Maturity Date [5, 7] — M_B
  - Coupon — C_B
  - Redemption amount [8] — R_B
  - CpnDates, the dates of the two coupon payments each year; and [9]
  - BcDates, the two books-closed dates corresponding to the coupon payment dates. [9]
- The nominal amount of the bond, in Rand — NOM

One further input is required, and is worth parameterising. This is:

- The number of decimal places to which prices are rounded [10] — PROUND

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[4] Y and C_B are expressed as percentages. So, for example, a yield of 12.52% means the number 12.52. Conversion to decimal numbers is handled explicitly in the text by division by 100 where necessary. Note that this conversion is applied to Y, but not to C_B. Y is a nominal annual rate, compounded semi-annually.

[5] S and M_B are supplied as calendar dates, which must be converted to a number of days from a fixed base date (for example, to Julian dates) so that arithmetic can be performed on them. Obviously, $S \leq M_B$.

[6] The information for the bond B must be read from a database of bonds.

[7] Several bonds have multiple redemption dates, on each of which a proportion of the total issue is redeemed. The GCH formula ignores this, and treats bonds as if they were to be fully redeemed on the given maturity date, M_B.

The convention for M_B in these cases is that it is the mid or average redemption date, as long as this coincides with a coupon date. In the past, affected bonds have been split into separate tranches, one for each redemption date, well before the first redemption date is reached: hence it has not been necessary to extend the convention to multiple redemption bonds which are in their redemption period. In this eventuality, or when the mid redemption date does not coincide with a coupon date, the fall-back position is that the bond becomes a price stock, and is traded on price by agreement between the counterparties to the trade.

Programmers need not concern themselves with this issue, as long as they can read the deemed redemption date, M_B, from a database. It would be wise, however, to establish from the Bond Exchange the accepted value of M_B for any problematic bonds.

[8] R_B is the capital redemption per R100 nominal of the Bond. It is invariably equal to 100; however, parameterising it allows for the possibility of bonds being redeemed at a premium or a discount, and avoids the need to hard-code a number in programs.

Note that implicit in the units of C_B and R_B is the fact that the GCH formula gives prices per R100 nominal of stock.

[9] The information required for the coupon payment and books-closed dates is the day and the month of each date. These are independent of the year, and remain the same for the whole life of a bond. The only exception is a coupon or books-closed date which falls on the last day of February. It is suggested that this is coded as dd=29, mm=2, with a correction being made in the logic for non-leap years. Where a date always falls on the 28th February (as in one of the coupon payment dates of the R150) the coding is dd=28, mm=2.

[10] The GCH formula specifies that PROUND is equal to 5. Hence, it prices bonds on R100 nominal, rounded to 5 decimal places, to get prices like 85.77155. It is conceivable that PROUND could change in the future; parameterising it allows for this, avoids hard-coding, and is also sometimes useful in studies when the discreteness introduced by rounding needs to be ignored.

2 OUTPUTS

The results to be calculated are:

- The accrued interest on the bond as at the settlement date: [11, 12] ***ACCRINT***{B,S}
- The all-in price of the bond at the yield on the settlement date: ***AIP***{B,S,Y}
- The clean price of the bond at the yield on the settlement date: ***CP***{B,S,Y}

If a nominal amount, NOM, of the bond has been traded or is being valued at the yield for the settlement date: [13]

- The interest consideration ***IntConsid***{B,S,NOM}
- The all-in consideration ***AllinConsid***{B,S,Y,NOM}
- The clean consideration ***CleanConsid***{B,S,Y,NOM}

The following two results are not part of the GCH formulation, but are easily available once the intermediate values have been found, are required for the differentials (duration, convexity, etc) of the bond price, and are useful for the inverse process of finding a yield from a price:

- The first partial differential of AIP with respect to the discount factor:

$$dAIP = \frac{\partial AIP\{B,S,Y\}}{\partial F}$$

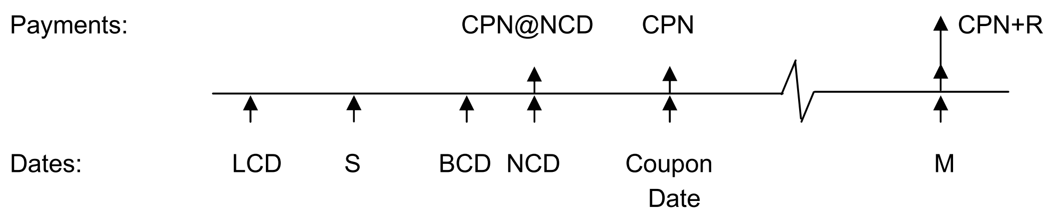
- The second partial derivative of AIP with respect to the discount factor:

$$dAIP = \frac{\partial^2 AIP\{B,S,Y\}}{\partial F^2}$$

[11] Note that accrued interest is independent of the Yield, Y. It may be positive or negative, the latter case obtaining while the bond is ex-interest for the settlement date.
[12] Results presented in ***bold italic*** type represent amounts rounded in terms of the GCH conventions. ACCRINT, AIP and CP all exist also in unrounded forms, which shall be needed - as for example in the calculation of ∂AIP/∂F. The unrounded forms are presented in normal type. The rounding can be counter-intuitive, as can be seen in the derivation of ***AIP*** in 4.3 to 4.6 below.
[13] The considerations were not explicitly mentioned in the original GCH formula, but through long practice they have come to be regarded as an integral part of it.

3 TIMING AND INTERMEDIATE RESULTS

The first step is to identify the position of S with respect to the bond's coupon payment and books-closed dates as illustrated below:



Define:

3.1 The last coupon date, LCD, as the most recent coupon payment date of the bond <u>on or before</u> S.

3.2 The next coupon date, NCD, as the next coupon payment date of the bond <u>after</u> S. (So, if S happens to coincide with a coupon payment date, LCD = S and NCD will be the coupon date in 6 months' time).

3.3 The books-closed date for the period, BCD, as the books-closed date relating to NCD. The BCD must be between LCD and NCD; it is generally 10 days before NCD.

3.4 The number of remaining coupon dates, N, <u>after</u> NCD but including the coupon date at M_B.[14]

Note that LCD, BCD and NCD are expressed as days from a common base date (the same base date as used for M_B and S).

Now calculate:

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[14] N can be easily calculated as:

$$N = \mathrm{Round}\left(\frac{M_B - NCD}{365.25/2}, 0\right)$$

The function ROUND[x,n] rounds the number x to n decimal places (with values of .5 * 10^{-n} and above being rounded up as usual). If x is negative, use the identity ROUND[-x,n] = -ROUND[x,n].

3.5 The cum/ex flag, CUMEX, which is a variable with the value 1 (for "TRUE") if the bond is cum interest on S and the value 0 (for "FALSE") if the bond is ex interest on S:[15]

$$\text{CUMEX} = 1 \quad \text{if} \quad S < \text{BCD}$$
$$= 0 \quad \text{if} \quad S \geq \text{BCD}$$

3.6 The number of days accrued interest, DAYSACC, as at S:

$$\text{DAYSACC} = S - \text{LCD} \quad \text{if} \quad \text{CUMEX} = 1$$
$$= S - \text{NCD} \quad \text{if} \quad \text{CUMEX} = 0$$

So a bond which is ex-interest has negative DAYSACC, and where the settlement date coincides with one of the bond's coupon payment dates, DAYSACC is zero.

3.7 The basic coupon amount payable on coupon payment dates, CPN:

$$\text{CPN} = C_B \;/\; 2$$

3.8 The coupon payable on NCD, CPN@NCD:

$$\text{CPN@NCD} = \text{CPN.CUMEX}$$

3.9 The semi-annual discount factor, F, corresponding to the Yield, Y:

$$F = \frac{1}{1 + Y/200}$$

The next four results differ according to whether the bond has more or less than six months to run to maturity. Bonds with six months or less to maturity are priced as money-market instruments. [16]A bond is deemed to have six months or less to maturity on and after its penultimate coupon payment date. That is, NCD=M_B and N=0.

[15] A bond goes ex interest in its last six months of life on the relevant books-closed date as usual. A purchaser of the bond in its final ex period will be buying the bond exdividend. However, the purchaser may be able to register the bond, up until its "Final Registration Date", and hence claim payment of the redemption amount in the normal way. The purchaser of a bond for settlement after its Final Registration Date will have to claim the redemption amount from the seller.

[16] The mathematical treatment of bonds in their last six months which follows will look unfamiliar to money-market practitioners. The method is, however, algebraically identical to the more usual formula, which can be found in Section 2 of the Appendix. The treatment here allows a single formula to be used for AIP and its derivatives in Section 4.

3.10 The broken-period, BP, measured in half-years:

$$BP = \frac{NCD - S}{NCD - LCD} \quad \text{if} \quad NCD \neq M_B$$

$$= \frac{NCD - S}{365/2} \quad \text{if} \quad NCD = MB$$

3.11 The broken-period discount factor, BPF:

$$BPF = F^{BP} \quad \text{if} \quad NCD \neq M_B$$

$$= \frac{F}{F + BP.(1 - F)} \quad \text{if} \quad NCD = M_B$$

The following two results are required only for dAIP and d2AIP which, as stated above, are not part of the GCH formula proper.

3.12 The first differential of BPF with respect to F, dBPF = $\partial BPF/\partial F$:

$$dBPF = \frac{BP.BPF}{F} \quad \text{if} \quad NCD \neq M_B$$

$$= \frac{BP.BPF^2}{F^2} \quad \text{if} \quad NCD = M_B$$

3.13 The second differential of BPF with respect to F, d2BPF = $\partial^2 BPF/\partial F^2$:

$$d2BPF = dBPF \,.\, \frac{BP - 1}{F} \quad \text{if} \quad NCD \neq M_B$$

$$= 2.dBPF \,.\, \frac{BP.BPF - F}{F^2} \quad \text{if} \quad NCD = M_B$$

4 RESULTS

4.1 Unrounded accrued interest: [17]

$$ACCRINT\{B,S\} = \frac{DAYSACC.C_B}{365} \qquad \text{Equation 1}$$

4.2 Rounded accrued interest:

$$\boldsymbol{ACCRINT\{B,S\}} = ROUND[ACCRINT,PROUND] \qquad \text{Equation 2}$$

4.3 Unrounded all-in price:

$$\begin{aligned} AIP\{B,S,Y\} &= BPF\left(CPN@NCD + CPN.\frac{F(1 - F^N)}{1 - F} + R.F^N\right) && \text{if} \quad F \neq 1 \\ &= CPN@NCD + CPN.N + R && \text{if} \quad F = 1 \end{aligned} \qquad \text{Equation 3}$$

4.4 Unrounded clean price:

$$CP\{B,S,Y\} = AIP - ACCRINT \qquad \text{Equation 4}$$

4.5 Rounded clean price:

$$\boldsymbol{CP}\{B,S,Y\} = ROUND[CP,PROUND] \qquad \text{Equation 5}$$

4.6 Rounded all-in price:[18]

$$\boldsymbol{AIP}\{B,S,Y\} = \boldsymbol{CP} + \boldsymbol{ACCRINT} \qquad \text{Equation 6}$$

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[17] Note that this is the "Actual/365" interest calculation convention.

[18] As mentioned above, the rounding of AIP is somewhat tortuous. The reason for this would appear to be that when the formula was first introduced, it replaced a convention based on the Hewlett Packard HP92 calculators. These, following American convention, produce a rounded clean price as their main output; (rounded) accrued interest was then added to give the (rounded) all-in price.

4.7 The differentials will be more easily presented if we define here some intermediate results:

$$dCPN = CPN.\frac{1-(N-N.F+1).F^{N}}{(1-F)^{2}} \quad \text{if} \quad F \neq 1$$

$$= CPN.\frac{N(N+1)}{2} \quad \text{if} \quad F = 1$$

Equation 7

$$dCPN = CPN.\frac{2-[N.(1-F).\{2+(N-1)(1-F)\}+2F].F^{N-1}}{(1-F)^{3}} \quad \text{if} \quad F \neq 1$$

$$= CPN.\frac{N(N^{2}-1)}{3} \quad \text{if} \quad F = 1$$

Equation 8

$$dR = N.R.F^{N-1}$$

Equation 9

$$d2R = N(N-1).R.F^{N-2}$$

Equation 10

4.8 First differential of AIP with respect to F:

$$dAIP = dBPF.\frac{AIP}{BPF} + BPF\left(dCPN + dR\right)$$

Equation 11

4.9 Second differential of AIP with respect to F:

$$d2AIP = d2BPF.\frac{AIP}{BPF} + dBPF.\left(\frac{BPF.dAIP - AIP.dBPF}{BPF^{2}} + dCPN + dR\right) + BPF.[d2CPN + d2R]$$

Equation 12

5 CONSIDERATION

Bonds are usually traded in lots of R1 million nominal, whereas the pricing above is for units of R100 nominal. The equations below detail the conversion from price to consideration, for a standard or odd-lot of R NOM nominal. The rounding is, again, counter-intuitive, although it will have no effect for standard lots.

5.1 Interest consideration:

$$\boldsymbol{IntConsid} = \text{ROUND}\left(\boldsymbol{ACCRINT}.\frac{\text{NOM}}{100},2\right) \qquad \text{Equation 13}$$

5.2 All-in consideration:

$$\boldsymbol{AllinConsid} = \text{ROUND}\left(\boldsymbol{AIP}.\frac{\text{NOM}}{100},2\right) \qquad \text{Equation 14}$$

5.3 Clean consideration:

$$\boldsymbol{CleanConsid} = \boldsymbol{AllinConsid} - \boldsymbol{IntConsid} \qquad \text{Equation 15}$$

6 DELTA, DURATION AND CONVEXITY

This section is presented for interest and completeness only. Note that implicit in theoretical definitions of duration and convexity is a different yield-to-price conversion than that of the GCH formula. For consistency of treatment the results below are based upon the GCH method.

6.1 Delta and Rands per Point

The delta of a bond is defined as the differential of its price with respect to its yield:

$$\text{Delta} = \frac{\partial AIP}{\partial Y} = -\frac{F^2}{200} \cdot \frac{\partial AIP}{\partial F}$$

Note that:

$$\frac{\partial CP}{\partial Y} = \frac{\partial AIP}{\partial Y}$$

The delta gives the Rand change in the all-in price per R100 nominal of a bond, for a unit change in the yield (eg from 14.75% to 15.75%); for a different change in yield:

$$\Delta P = \text{Delta} \, . \, \Delta Y$$

where ΔP is the change in price and ΔY the change in yield. It is conventional to use a ΔY of .01 (which is equal to one point, given that market yields are expressed as percentages) and to find the price change per R1m nominal of stock. In this case the "Rands per Point" per R1m nominal is:

$$\text{Rands per Point} = 100.\text{Delta}$$

The delta is a negative figure (because an increase in yield gives a decrease in price); the sign is usually dropped for the Rands per Point.

6.2 Modified Duration

The modified duration of a bond, measured in years, is

$$\text{Dmod} = -100.\frac{\partial AIP}{\partial Y} \Big/ AIP$$

6.3 Duration

The duration of a bond, measured in years, is:

$$Dur = DMod / F$$

6.4 Second Differential and Convexity

The second differential of the all-in price with respect to the yield is:

$$\frac{\partial^2 AIP}{\partial Y^2} = \left(\frac{\partial AIP/\partial F . F^3}{2} + \frac{\partial^2 AIP/\partial F^2 . F^4}{4} \right) \Big/ 10000$$

where the units are Rands/Percent/Percent per R100 nominal; or, equivalently (the factors cancel out), Rands/Point/Point per R1m nominal.[19] Another common measure which is related is Convexity:

$$Conv = \frac{10000}{AIP} \cdot \frac{\partial^2 AIP}{\partial Y^2}$$

[19] Note that this measure is also sometimes given in Rands/Point/100 Points per R1m nominal.

7 PRICE TO YIELD

7.1 Specification

It is often necessary, given a clean or all-in price of a bond for a settlement date, to calculate the yield that this implies for settlement on that date. This is known as an "implied yield".

The GCH formula does not specify the terms of this conversion. The following rule describes majority market practice:

The implied yield of a bond is that yield which produces an unrounded all-in price equal to the target all-in price; the yield is then rounded to 5 decimal places.

This definition is incomplete, in that it does not specify the precision to which prices are "equal". In practice, it means to a "large number" of decimal places; the Exchange's capture system uses 11 decimal places. However, under this approach, no matter how many decimal places are used, it is always possible to find examples where extra precision will change the result by 1 in the 5th decimal place.

An alternative is to recognise that, since the final result is to be rounded to 5 decimal places, it is sufficient to locate the true yield only within a range, all numbers in which round to the same unique 5 decimal place value. This approach, which is used in the example algorithm below, will always converge to a single result, given sufficient precision.

Note that, because It is not generally possible for an implied yield to produce either a rounded or unrounded price exactly equal to the target price, implied yields must often be regarded as "for information only". However, given that the maximum difference between the implied and true yields is ±½ in the 6th decimal place, i.e. 1/2000th of a point, and that the Rands per point on most long bonds is ± R500 per R1m nominal, the error which can arise is of the order of 25c per R1m nominal for these bonds.

The method by which implied yields are found is not relevant. Therefore, the remainder of this section, which describes a particular algorithm, is recommendatory in nature only.

7.2 Example Algorithm

The formula connecting price and yield, as given in Equation 3 above, generally cannot be inverted - ie. it cannot be solved algebraically to give an expression for F (and hence for Y). Iterative numerical methods must be used instead. Since the calculation is frequent, it is worth using a method which converges quickly. The algorithm below uses Bailey's method, an extension of the well-known Newton Raphson method. This method usually converges within 2-3 iterations, as compared with Newton-Raphson's 4-5; it also has the virtue of converging from a wider range and being less likely to produce wild results.[20]

The convergence test used below assumes that the difference between successive trial values of the yield never increases. Having found a new trial value, a range is constructed with this value at the centre, and the previous trial value at one end. If the values at both ends of the range round to the same result, convergence has taken place.

As stated above, implied yields are quoted to 5 decimal places. We therefore introduce another parameter, the number of places that yields (expressed as percentages) are rounded to: YROUND = 5.

A[21] further four parameters controlling the iterative process below are required. Recommended values for these are:

First guess for implied yield:	Y_0 = 10%
Iteration limit:	ItnLim = 5
Minimum allowable yield:	MinY = -67%
Maximum allowable yield:	MaxY = 200%

We wish to specify the calculation of the function:

ImplY{B,S,AIP}

giving the rounded yield, **ImplY**, which produces an unrounded all-in price equal to the target all-in price, AIP, for the bond B on settlement date S.[22]

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[20] The Newton Raphson method can be treated as a special case of Bailey's method. If the second differential, d2AIPi in 7.2.1.2 is set to zero, the Newton Raphson method results. This may be preferred in some cases, because it avoids the need to calculate complicated second derivatives.

[21] These may be updated as experience dictates and with changing circumstances. For example, the first guess can be chosen close to the average level of yields along the yield curve; alternatively, a value of zero will (almost always) converge to the same result, in 1 or 2 extra iterations. Similarly, the iteration limit can be increased if the only reason for non-convergence is insufficient iterations.

[22] If the given price is a clean price, CP, convert it to the corresponding all-in price, using (an inversion of) Equation 4:

AIP = CP + ***ACCRINT***

Note that the prices must be for units of R100 nominal, but will not necessarily be rounded at the 5 decimal places of PROUND.

7.2.1 Perform successive iterations with i=0,1,2, ... ItnLim as follows:

7.2.1.1. Find:

$$AIP_i = AIP\{B,S,Y_i\},$$

$$dAIP_i = \partial AIP\{B,S,Y_i\}/\partial F$$

and $$d2AIPi=\partial^2 AIP\{B,S,Yi\}/\partial F^2$$

from Equations 3, 11 and 12 respectively.[23]

7.2.1.2. Find the next trial value, Y_{i+1} from

$$diff = AIP_i - AIP$$

$$F_{i+1} = F_i - \frac{diff}{dAIP_i - \frac{diff.d2AIP_i}{2.dAIP_i}}$$

$$Y_{i+1} = 200 / F_{i+1} - 200$$

7.2.1.3. Check for values out of range:

If $Y_{i+1} < MinY$ or $Y_{i+1} > MaxY$ go to Step 7.2.2.

7.2.1.4. Test for convergence:

Find rounded value of previous trial:

$Prev = ROUND[Y_i,YROUND]$

Find rounded value of opposite end of range:

$Opp = ROUND[2Y_{i+1} - Y_i,YROUND]$

[23] Note that when i=0, the value Y_i is given by the parameter Y_0 defined above in Section 7.2

If Prev = Opp, convergence has taken place. Set

Imply = Opp

and exit.

If not, increment i and return to Step 7.2.1.1.

If ItnLim has been reached, the process has failed to converge. Proceed to Step 7.2.2

7.2.2 If the process has failed to converge, or has produced wild results, an error return such as "Null" must be returned. No further processing is possible here.

8 PRECISION

Except where otherwise stated, all intermediate results are calculated and held to full precision. Given that a price contains 7-8 significant digits, and an implied yield 6-7 significant digits, single precision calculations (which are correct to 6-7 significant digits) are not sufficient. Hence all calculations and intermediate results should be carried to at least 11 significant digits; and preferably to full double precision (15-16 significant digits).

9 EXAMPLES

In the two examples which follow, references are to Section numbers above. An ellipsis (...) to the right of a number indicates that it is not shown to its full precision. Numbers without the ellipsis are exact.

Example 1

Find price information and the considerations for the purchase of an odd-lot of R1,500,000 nominal of R186 stock, at a yield of 7.5%, for settlement on 26 August 2005.

	DESCRIPTION	SYMBOL	DATUM	
1.	Inputs			
	Yield-to-maturity	Y	7.5	
	Settlement date	S	26 August 2005	
	Bond	B	R186	
	The following information on the bond, R150, is read from a database of bonds:			
	Maturity Date	M	21-Dec-2026	
	Coupon	C	10.5	
	CpnDates	1st	21/06	
		2nd	21/12	
	BcDates	1st	11/06	
		2nd	11/12	
	The following two parameters are the same for all bonds			
	Redemption amount	R	100	
	Price rounding decimal places	PROUND	5	
2.	Results See section 2 of the text for an introduction to the results to be calculated			
3.	Timing and Intermediate Results			
3.1.	Last coupon date	LCD	21-Jun-05	
3.2.	Next coupon date	NCD	21-Dec-05	
3.3.	Books-closed date	BCD	11-Dec-05	
3.4.	No. of remaining coupon dates	N	42	
3.5.	Cum-ex flag	CUMEX	1	
3.6.	Days accrued interest	DAYSACC	66	
3.7.	Basic coupon amount	CPN	5.25	
3.8.	Coupon payable on NCD	CPN@NCD	5.25	
3.9.	Semi-annual discount factor	F	0.963855421686747	…

	DESCRIPTION	SYMBOL	DATUM	
3.10.	Broken-period	BP	0.639344262295082	…
3.11.	Broken-period discount factor	BPF	0.97673802761755	…
3.12.	First differential of BPF	dBPF	0.647889548237297	…
3.13.	Second differential of BPF	d2BPF	-0.242427523582234	…
4.	Results			
4.1.	Unrounded accrued interest	ACCRINT	1.8986301369863	…
4.2.	Rounded accrued interest	***ACCRINT***	1.89863	
4.3.	Unrounded all-in price	AIP	133.547091364729	…
4.4.	Unrounded clean price	CP	131.648461227743	…
4.5.	Rounded clean price	***CP***	131.64846	
4.6.		***AIP***	133.54709	
4.7.		dCPN	1862.62309856317	…
		d2CPN	45079.3564011142	…
		dR	928.403928848386	…
		d2R	39491.9821233882	…
4.8.	First differential of AIP	dAIP	2814.68664663936	…
4.9.	Second differential of AIP	d2AIP	86187.4503185668	…
5.	Consideration			
5.1.	Interest consideration	IntConsid	28,479.45	
5.2.	All-in consideration	AllInConsid	2,003,206.35	
5.3.	Clean consideration	CleanConsid	1,974,726.90	
6.	Delta, Duration and Convexity			…
6.1.	Delta	Delta	-13.0744625769284	…
	Rands per Point	Rands per Point	R 1,307.45	…
6.2.	Modified Duration	Dmod	9.79015150634829	…
6.3.	Duration	Dur	10.1572821878364	…
6.4.	Second differential		1.98567065431985	…
	Convexity	Conv	148.686926388895	…

Example 2

Find the implied yield of the R186 bond, given that its all-in price for settlement on 26 August 2005 is 95.123456789

	DESCRIPTION	SYMBOL	DATUM					
7.	Price to Yield							
	Parameters							
	Yield rounding decimal places	YROUND	5					
	First guess for implied yield	Y_0	10.0					
	Iteration Limit	ItnLim	5					
	Minimum allowable yield	MinY	-67.0					
	Maximum allowable yield	MaxY	200.0					
	Bond	B	R186					
	Settlement Date	S	21-Dec 2026					
	Target all-in price	AIP	95.123456789					
					ITERATION			
7.2.1	Iterate		I=0		I=1		I=2	
	Yield to maturity	Y_i	10		11.34241977	…	11.34459412	…
	Discount factor	F_i	0.9523809524	…	0.94633155152	…	0.94632181550	…
7.2.1.1	All-in price	AIP_i	106.23956578	…	95.13982801	…	95.12345679	…
	1st differential of AIP	$dAIP_i$	1998.7921662	…	1681.7404217	…	1681.2806705	…
	2nd differential of AIP	$d2AIP_i$	57984.112556	…	47229.418148	…	47213.938653	…
7.2.1.2	Distance from target AIP	diff	11.116108994	…	0.016371219	…	0.0000000001	…
	Next discount factor	F_{i+1}	0.946331552	…	0.946321816	…	0.946321816	…
	Next trial yield	Y_{i+1}	11.34241977	…	11.34459412	…	11.34459412	…
7.2.1.3	Out of range?		No		No		No	
7.2.1.4	Test for convergence							
	Rounded previous trial	Prev	10.00000		11.34242		11.34459	
	Rounded opposite end	Opp	12.68484		11.34677		11.34459	
	Done?		No		No		Yes	
	Rounded implied yield	***ImplY***					11.34459	

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APPENDIX

THE ORIGINAL GCH FORMULA

Sections 1 and 2 of this Appendix give the GCH formula as it was originally specified, with just one or two changes.

1. BONDS WITH MORE THAN 6 MONTHS TO REDEMPTION

$$\textbf{Unrounded All-in Price} = V_i^{\frac{d_1}{d_2}} \left(\frac{1}{2} g \left(a_n^i + e \right) + 100V_i^n \right)$$

Where	d_1	=	number of days from settlement date to next interest date
	d_2	=	number of days from last to next interest date or from settlement date to next interest date if settlement falls on an interest date
	I	=	yield at which bond trades, as a percentage
	V_i	=	1 / (1 + I/200)
		=	present value of 1 payable in 6 months' time
	g	=	coupon as a percentage
	n	=	number of complete six month periods from next interest date to redemption date
	a_n^i	=	$(1 – V_i^n) / (I/200)$
		=	present value of an annuity of 1 per six months, payable in arrears
		=	1 if the bond is cum and 0 if ex

Accrued Interest = $\dfrac{d_2e - d_1}{365} \times g$

Clean Price = All-in price - Accrued interest

Note:

1. Rounding convention: Clean price is rounded to 5 decimal places, accrued interest then rounded to 5 decimal places and added back to the clean price to arrive at the all-in price

2. Bonds are considered to be cum interest on a coupon date

2. BONDS WITH LESS THAN 6 MONTHS TO REDEMPTION

$$\text{Unrounded All-in price} = \frac{100 + e \times \frac{g}{2}}{1 + \frac{d_1}{365} \times \frac{i}{100}}$$

with definitions as before

Accrued interest = as for longer bonds

Rounding as with longer bonds.

3. PRICE TO YIELD

The implied yield of a bond is the yield which produces an unrounded all-in price equal to the target all-in price, rounded to 5 decimal places.

4. PRECISION

All calculations and intermediate results should be carried to at least 11 significant digits; and preferably to full double precision (15-16 significant digits).