Volatility is a measure of the risk or uncertainty and plays important role in the financial markets

JSE

MARKET DATA

Indices

SAVI Squared

Johannesburg
Stock Exchange

Variance Futures are contracts that obligate the holder to buy or sell variance at a predetermined variance strike at a specified future time. Variance has the interesting property of directly increasing with volatility. Hence, a direct exposure to volatility is therefore afforded by having a position in variance futures.

Volatility is a measure of the risk or uncertainty and it has an important role in the financial markets. Volatility is defined as the variation of an asset's returns – it indicates the range of a return's movement. Large values of volatility mean that returns fluctuate in a wide range – in statistical terms, the standard deviation is such a measure and offers an indication of the dispersion or spread of the data[1].

Volatility forms an integral part of the Black-Scholes-Merton option pricing model [BS 73]. Even though the model treats the volatility as a constant over the life of the contract, these three pioneers knew that volatility changes over time. As far back as 1976, Fischer Black wrote [Bl 76]

"Suppose we use the standard deviation … of possible future returns on a stock … as a measure of volatility. Is it reasonable to take that volatility as constant over time? I think not."

Black&Scholes defined volatility as the standard deviation because it measures the variability in the returns of the underlying asset [BS 72]. They determined the historical volatility and used that as a proxy for the expected or implied volatility in the future. Since then the study of implied volatility has become a central preoccupation for both academics and practitioners [Ga 06].

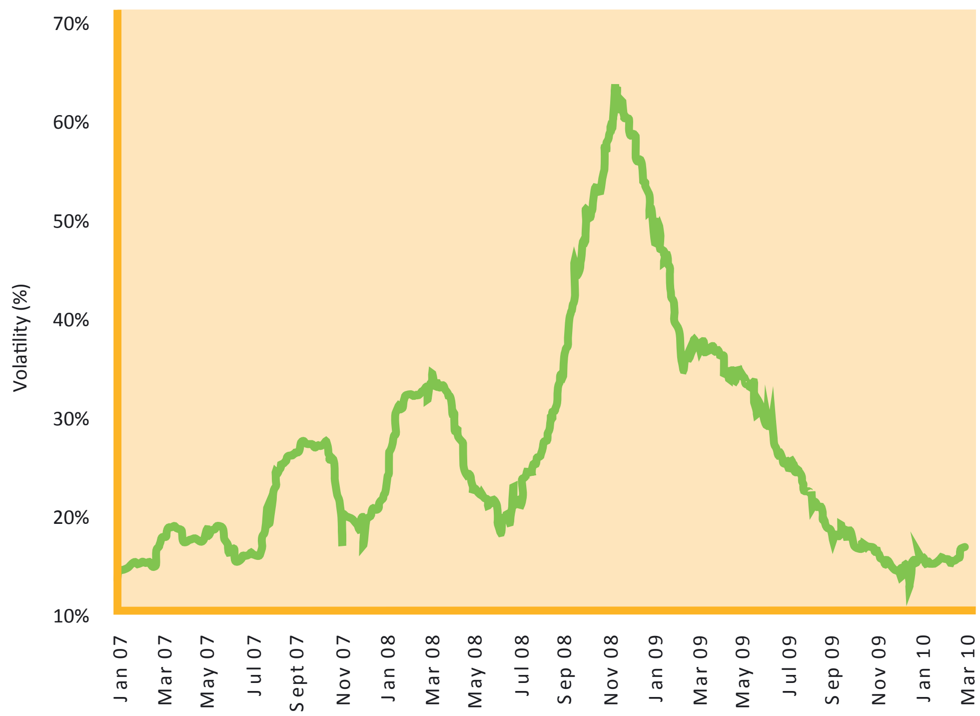


Figure 1. Three month historical rolling volatility for the JSE/FTSE Top 40 index from January 2007 to April 2010.

1 The standard deviation is one of the fundamental elements of the Gauss or normal distribution curve – it describes the width of the famous bell curve.

Figure 1 shows a plot of the 3 month historical volatility for the JSE/FTSE Top 40 index since January 2007 using daily data. It is clear that volatility is not constant.

Volatility is, however, statistically persistent, i.e., if it is volatile today, it should continue to be volatile tomorrow. This is also known as volatility clustering and can be seen in Figure 2. This is a plot of the 4 logarithmic returns[2] of the Top 40 index since June 1995 using daily data.

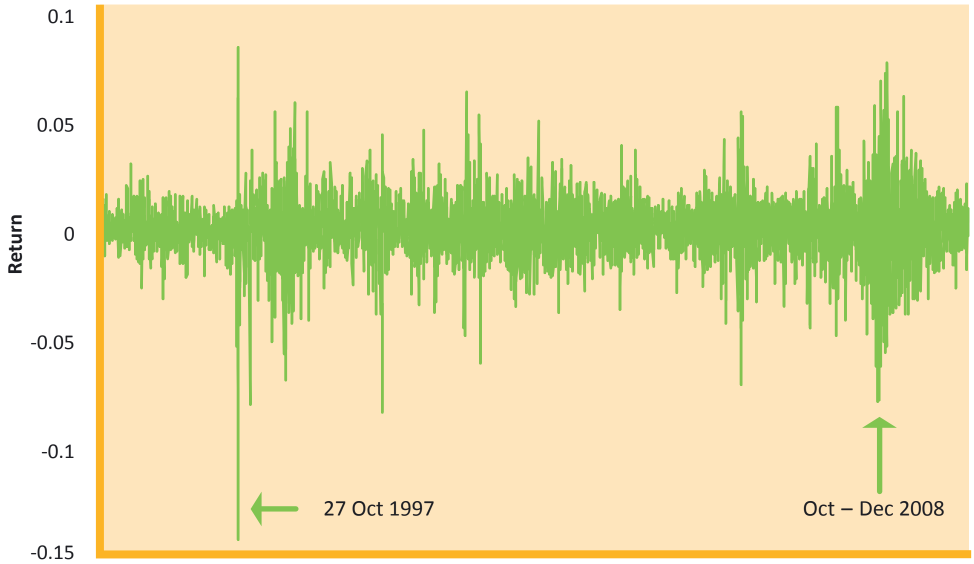


Figure 2. JSE/FTSE Top 40 daily logarithmic returns from June 1995 to September 2008. Note the 13.3% negative return on 28 October 1997 – start of the Asian crises.

But, what is this "volatility"? As a concept, volatility seems to be simple and intuitive. Even so, volatility is both the boon and bane of all traders – **you can't live with it and you can't really trade without it**. Without volatility, no trader can make money! Most of us usually think of "choppy" markets and wide price swings when the topic of volatility arises. These basic concepts are accurate, but they also lack nuance.

How do we define the volatility (standard deviation) as being good, or acceptable, or normal? By many standards, a large standard deviation indicates a non-desirable dispersion of the data, or a wide (wild) spread. It is said that such a phenomenon is very volatile – such phenomena are much harder to analyse, or define, or control. Volatility also has many subtleties that make it challenging to analyse and implement [Ne 97]. The following questions immediately come to mind: is volatility a simple intuitive concept or is it complex in nature, what causes volatility, how do we estimate volatility and can it be managed?

The management of volatility is currently a topical issue. Due to this, many institutional and individual investors have shown an increased interest in volatility as an investment vehicle.

2 Logarithmic returns of share index data are normally distributed.

Volatility as an Investable Asset Class

Apart from its prominent role as financial risk measure, volatility has now become established as an asset class of its own. Initially, the main motive for trading volatility was to manage the risk in option positions and to control the Vega exposure independently of the position's delta and gamma. With the growth of the volatility trading segment, other market participants became aware of volatility as an investment vehicle [HW 07]. The reason is that volatility has peculiar dynamics:

- It increases when uncertainty increases.
- Volatility is mean reverting – high volatilities eventually decrease and low ones will likely rise to some long term mean.
- Volatility is often negatively correlated to the stock or index level.
- Volatility clusters.

How can investors get exposure to volatility? Traditionally, this could only be done by taking positions in options, and then, by delta hedging the option market exposure.

Delta hedging, however, is at best, inaccurate due to the Black&Scholes assumptions like continuous trading. Taking a position in options, therefore, provides a volatility exposure (Vega risk) that is contaminated with the direction of the underlying stock or index level.

Variance swaps emerged as a means of obtaining a purer volatility exposure. These instruments took off as a product in the aftermath of the Long Term Capital Management (LTCM) meltdown in late 1998 during the Asian crises. Some additional features are

- variance is the square of volatility;
- simple payoffs; and
- simple replication via a portfolio of vanilla options.

These are described in the next few sections.

The "SAVI Squared" product

"SAVI Squared" or variance futures (also called variance contracts) are equity derivative instruments offering pure exposure to daily realised future variance. Variance is the square of volatility (usually denoted by the Greek symbol σ^2).

At expiration, the buyer receives a payoff equal to the difference between the annualised variance of logarithmic stock returns and the rate fixed at which he bought it. The fixed rate (or delivery price) can be seen as the fixed leg of the future and is chosen such that the contract has zero present value.

In short, a **variance future is not really a future at all but a forward contract on realised annualised variance**. A long position's payoff at expiration is equal to [DD 99]

Equation {1}

$$VNA\left[\sigma_R^2 - K_{del}\right]$$

where VNA is the Variance Notional Amount, σ_R^2 is the annualised non-centered realised variance of the daily logarithmic returns on the index level and Kdel is the delivery price. Note that VNA is the notional amount of the contract in Rand per annualised variance point.

The holder of a "SAVI Squared" at expiration receives VNA Rands for every point by which the stock's realised variance has exceeded the variance strike price.

A capped variance future is one e the realised variance is capped at a predefined level. From Eq.{1} we then have

$$VNA\left[\min\left(cap,\sigma_R^2\right)-K_{del}\right]$$

The realised variance is defined by

$$\sigma_R^2=\frac{252}{n}\sum_{i=1}^{n}\left[\ln\left(\frac{S_i}{S_{i-1}}\right)\right]^2$$

with S_i the index level and *n* the number of data points used to calculate the variance [BS 05].

If we scrutinised Eq.{2}, we see that the mean logarithmic return is dropped if we compare this equation with the mathematically correct equation for variance. Why? Firstly, its impact on the realised variance is negligible.

Secondly, this omission has the benefit of making the payoff perfectly additive[3]. Another reason for ditching the mean return is that it makes the estimation of variance closer to what would affect a trader's profit and loss. The fourth reason is that zero-mean volatilities/variances are better at forecasting future volatilities. Lastly *Figlewski* argues that, since volatility is measured in terms of deviations from the mean return, an inaccurate estimate of the mean will reduce the accuracy of the volatility calculation [Fi 94]. This is especially true for short time series like 1 to 3 months (which are the time frames used by most traders to estimate volatilities).

Note, historical volatility is usually taken as the standard deviation whilst above we talk about the variance. The question is: why is standard deviation rather than variance often a more useful measure of variability? While the **variance** (which is the square of the standard deviation) **is mathematically the "more natural" measure of deviation**, many people have a better "gut" feel for the standard deviation because it has the same dimensional units as the measurements being analysed. Variance is interesting to scientists, because it has useful mathematical properties (not offered by standard deviation), such as perfect additivity (crucial in variance swap instrument development). However, volatility is directly proportional to variance.

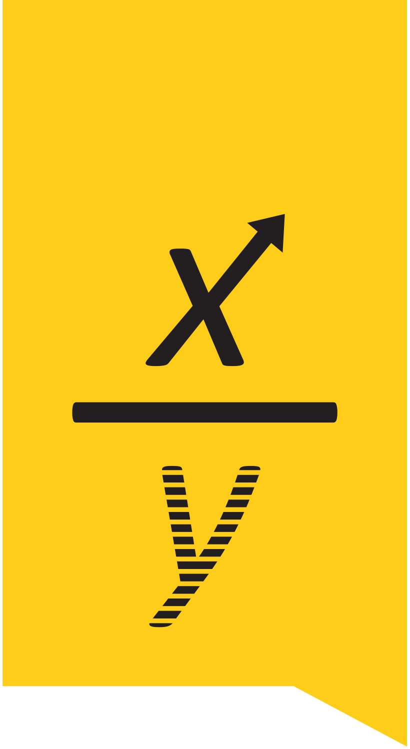
Variance Future Pricing in Theory

The price of the variance future per variance notional at the start of the contract is the delivery variance Kdel.

So how do we find the delivery price such that the future is immune to the underlying index level? Carr and Madan came up with a static hedge by considering the following ingenious argument [CM 98]: We know that the sensitivity of an option to volatility, Vega, is centered (like the Gaussian bell curve) around the strike price and will thus change daily according to changes in the stock or index level. We also know that the higher the strike, the larger the Vega. If we can create a portfolio of options with a constant Vega, we will be immune to changes in the stock or index level – a static hedge. Further, by induction, it turns

"If we can create a portfolio of options with a constant Vega, we will be immune to changes in the stock or index level"

3 Suppose we have the following return series: 1%, 1%, -1%, -1%. Using variance with the sample mean, the variance over the first two observations is 0, and the variance over the last two observations is zero. However the total variance with sample mean over all 4 periods is 0.01333%, which is clearly not zero. If we assumed the sample mean to be zero, we get perfect variance additivity



out that the Vega is constant for a portfolio of options inversely weighted by the square of their strikes. This hedge is independent of the stock level and time.

From the previous argument we deduce that the fair variance of a variance futures contract[4] is the value of a static option portfolio, including long positions in out-the-money (OTM) options, for all strikes from 0 to infinity. The weight of every option in this portfolio is the inverse square of its strike. Demeterfi and Derman et. al. discretised Carr and Madan's solution and set out all the relevant formulas and this will be the subject of the technical note [DD 99].

Pricing in Practice

Here are some practical hints to consider when estimating the implied variance, Kvar, through the portfolio of options:

- The chosen strikes have fix increments that are not infinitesimally small. This introduces a discretisation error[5] in the approximation of the fair variance. The smaller the increments the smaller the error. The JSE will use an increment of 10 index points in the valuation of variance futures on indices.
- A strike range is chosen and is thus also finite. This introduces a truncation error. This range is heavily dependent on the range of volatilities available. Note that the fair variance accuracy is more sensitive to the strike range, than the strike increments. In other words, the accuracy of the fair variance is more prone to truncation errors. The JSE publishes volatility skews for a strike range of 70% to of 130%. The strike range used will thus be from 70% moneyness to 130% moneyness.
- For a small number of symmetrical options (< 20) we have that the lower the put bound strike the more over-estimated the fair variance. This over-estimation is less pronounced the higher the number of symmetrical options.
- For a low put bound strike (< 40%) we have that the higher the number of symmetrical options, the more underestimated the fair variance. This under-estimation is considerably less pronounced the higher the left wing bound.

Finding the optimal strike increments (discretisation) and the strike range (minimise truncation errors) are a trade-off between over and under estimating the fair variance. [Ji 05]

Mark to Market

Any time, *t*, during the life of a SAVI Squared, expiring at time *T*, the fair value of the variance future, can be decomposed into a realised variance part (the already crystallised variance), and an implied variance part (the variance part that must still be crystallised or future/fair variance). Because variance is additive the *price of the variance future of 1 variance notional* at time *t* (or the price of a new future maturing at time *(T–t)*), is

$$V_{mtm} = \frac{t}{T}\sigma_R^2 + \frac{T-t}{T}K_{\text{var}}(t,T)$$

4 The fair price of a variance swap, given that it is the price of variance in the future, it is also referred to as the fair value of future variance.

5 We implicitly use the term error, here and not uncertainty. This is so because an error implies that the theoretical true value is known, whereas uncertainty does not.

With Kvar*(t,T)* the implied variance at time *t*. At contract initiation the value will be derived from 100% implied variance. This means that at *t=0*

$$K_{del} = K_{var}(t,T)$$

However, as the contract moves closer to expiry realised variance will play an increasingly important role. This is shown in Figure 3 below. Variation margin is thus the difference between the value of the variance future today and what it was yesterday as given by Eq. {3}.

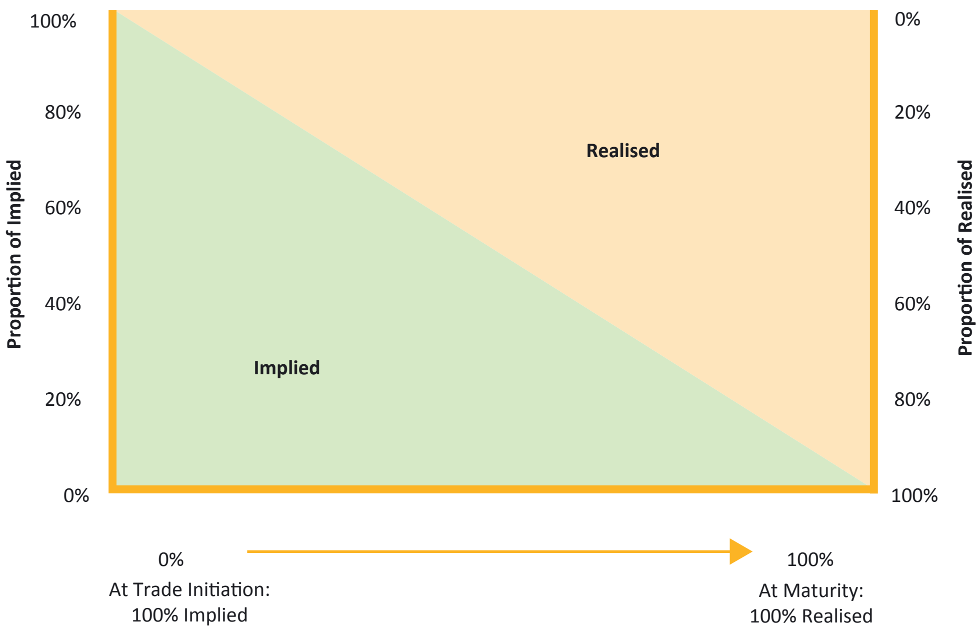


Figure 3: As time passes, realised variance start to play the dominant role in the value of a variance future.

Initial Margin Requirements

The initial margin requirement is determined in using the underlying's historical data to determine a volatility of volatility. The parameter λ is calculated as the maximum change over the historical 90 – 95% strike, volatilities. λ can then be thought of as the parameter that measures the expected one day volatility of volatility inferred from the historical volatility skews. The initial margin requirement *(IMR)* per contract for a variance future is given by (1-day value at risk measure)

Equation {4}

$$IMR = \Delta P_{\max} = VPV * [2 * \lambda * \sqrt{K_{var}} + \lambda^2].$$

Here, Kvar is the delivery variance and VPV the Variance Point Value. Note, the VPV is a quantity similar to the current R10 per point used for index futures. It is currently set at R1 for SAVI Squared contracts.

Most traders trading a variance future usually start with a Vega Rand amount they want to hedge – this might be due to a book of options. We then define the Variance Notional Amount [Wi 08]

Equation {5}

$$VNA = \frac{VA}{2\sqrt{K_{\text{var}}}} = C * VPV$$

with *VA* the Vega amount to be hedged in Rand. Here, Kvar is in absolute terms e.g., if the volatility is 30% then Kvar = 30^2 = 900. From this

Equation {6}

$$C = \frac{VNA}{VPV} = \frac{VA}{2\sqrt{K_{\text{var}}}\,VPV}.$$

Eq.{6} is used to determine the number of contracts, C, to trade at the initiation of the Variance futures life.

Note that the number of contracts defined by Eq. {6} is only applicable when trading on the day of listing. If it is required to trade in a different listed variance future (a contract that was listed a time *t* ago that matures at time *T*), e.g., rolling from one contract into another, the number of equivalent contracts to trade C*, can be shown to be

Equation {7}

$$C^* = \frac{VA}{2\sqrt{K_{\text{var}}}\,\frac{T-t}{T}}$$

With Kvar the implied variance for the remaining time *T–t*.

From Eq.{3} and Figure 3 we deduce that the risk in a variance future dwindles as we approach the expiry date – there is more certainty in the outcome because of the realised variance part. To arrive at the initial margin for the contact, we price up the initial margin for a new contract that has a similar time to expiry as the existing

SAVI Squared contract and then adjust the value down according to the percentage time elapsed. This adjustment is done only once a month. As an example we look at both a 3 month and a 6 month variance future contracts. By applying Eq. {4}, we calculate the Initial Margin Requirements. The initial margins will then reduce in accordance with the table below

Indicative Initial Margin (IRM) requirements per contract		
Time remaining to expiry	3month IMR	6month IMR
1m	R 48	R 24
2m	R 88	R 44
3m	R 120	R 60
4m		R 75
5m		R 85
6m		R 93

Table 1: Example of IMRs for new and listed contracts.

Table 1 is understood if we look at the following example: if a trader goes long the 6 month contract 3 months into its life, the margin requirement is half that of a new 3 month contract. From the table we deduce that a new 3 month contract's IMR is R120. If the trader goes long the existing 6 month contract, the IMR will be R60. The reasoning for this is that if one were to use Eq. (7) to calculate the number of contracts required one would find that you would need twice the number of 6 month contracts to offset the corresponding 3 month contract.

Profit and Loss

To determine the profit and loss for a "SAVI Squared" contract we return to Eq.{1} and {3}. This gives

Equation {8}

$$PL = VNA\left[V_{mtm} - K_{\mathrm{var}}\right] = C * VPV\left[V_{mtm} - K_{del}\right]$$

where Vmtm is the daily mark-to-market variance level as calculated using Eq.{3} and is set on a daily basis. Kdel is delivery price or strike at which you traded set at initiation of the contract. This *PL* is paid over the life of the contact as your daily gains and losses are calculated at the end of each day.

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Appendix A:

Realised Variance will be calculated as follows $RV = 10000 \times \frac{252}{n} \sum_{i=1}^{n} \left[\ln \left(\frac{S_i}{S_i - 1} \right) \right]^2$

Where
i = each observation trade date during contract life
n = number of observed trading days during life of contract; note this excludes the listing date
S_0 = the official closing level of the underlying index on the listing date
S_i = the closing level of the underlying index future on thei-th observation date
S_n = the closing level of the underlying index on the expiration date

SAVI Squared Contract Specifications	
Name	SAVI-Squared
Future Contract	Variance Future Contract
Underlying Instrument	FTSE/JSE TOP40 Index
Codes	e.g. Dec10 SAV3 (3 month future contract expiring in December of 2010) or Dec10 SAV6 (6 month future contract expiring in December of 2010)
Listing Programme	On every ALSI Future Expiry date. 3 and 6 month contract to be listed.
Expiry Dates & Times	13h40 on the 3rd Thursday of Mar, Jun, Sep & Dec (for the previous business day if a public holiday)
Mark-to-market process	Closing mark-to-market contract calculated by JSE as time weighted sum of Realised Variance and Implied Variance (for more information on calculation of Implied Variance please refer to the website)
Expiry Valuation Method	Realised Variance as calculated by the JSE over the contract period (formula in Appendix A)
Quotations	Variance point to 2 decimals
Minimum Quotation Movement	R0.01 (0.01 Variance Point)
Variance Point Value	Fixed R1 per point
Variance Cap	2,5 times the initial corresponding Volatility of the contract's listing Variance
Settlement	Cash
Margin	Can be found on website: www.jse.co.za
Trading Times	Weekdays between 08h30–17h30

Further information on the contract and how the valuation is done can be found on the website: www.jse.co.za. Should you have any queries regarding SAVI, please contact us on +27 11 520 7000 or email equityderivatives@jse.co.za.

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