MONEY MARKET SUBCOMMITEE(MMS)
FLOATING RATE NOTE PRICING SPECIFICATION

This document outlines the use of the *margin discounting methodology* to price *vanilla money market floating rate notes* as endorsed by the South African Money Market Subcommittee. The pricing of listed floating rate notes will be covered by the JSE's Floating Rate Note pricing specification document.

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Floating Rate Note Pricing

1. Introduction

Like standard fixed bond-coupon bonds, FRN's are debt instruments that make periodic coupon payments. However, for FRN's each payment amount is dependent on the (uncertain) level of a pre-specified reference index. The rate reset date is at the beginning of the coupon period, and a spread is quite commonly added to the observed reference rate. The spread reflects the credit risk of the issuer, the maturity of the note and the liquidity, demand for that type of the instrument in the market.

In international markets, the most common benchmark is LIBOR. In South Africa, the most popular benchmark is 3month JIBAR. The reset of the coupon rate is usually quarterly in South Africa; however, monthly and semi-annually resets also exist. Coupon payments are calculated at the beginning of each coupon period, and paid in arrears.

2. Summary

Floating rate notes are priced on All-in price per R100 nominal. The following methodology is used to determine the All-in price of the FRN:

- Generate coupon payment date.
- Determine the mid swap zero rate corresponding to the coupon date.
- Calculate the forward rate from the calculated discount factor for each future coupon date using the mid swap zero rates.
- Predict the future coupons using the forward curve plus the issue spread.
- Discount each coupon and principal back to the present using the swap zero curve plus the current market spread.*(these are constructed using the product of the **forward period discount factors** to ensure that the FRN always prices to par on the date of issue and whereby the issue spread is equal to the market spread)*

The model has the following assumptions:

- FRN trades on price
- Coupon rates varies (due to implied future Jibar rates)
- Coupon periods are exact
- the zero rate at each coupon date are **linearly interpolated** from the swap zero curve
- Discount functions (**df's)** are not interpolated, but calculated using the zero rates at each coupon dates.
- Predict the future Jibar rates off the *implied forward curve* to calculate the future coupons
- Discount off the *mid swapzero curve plus the market spread* to calculate the price of the FRN
- Modified following date convention
- Structured FRN's are no covered by this model.

3. Method of Valuation

The methodology described in this document assumes quarterly payments. In addition we assume that interest is paid on coupon dates, unless the coupon date is on a Saturday, Sunday or public holiday, in which case the interest is paid on the next business day and the coupon includes these extra days.

FRN's are commonly evaluated via what is known as the *"discount margin method"*. This involves discounting all the cash flows back to their present value, to find the correct price for a given estimated yield. The process is similar to the calculation for fixed bond-rate securities, except with FRN's you do not know what the exact coupon payments will be. You must therefore make a prediction about where Jibar will be over the life of the FRN.

The mid Swap Zero curve is used to predict what the market believes the future Jibar rates will be. These forward rates are implied from swap zero rates. You then use these rates (plus the issuance spread) to calculate the future coupons as well discount the future coupons (*using the zero rate plus the market spread*) and principal back to present.

The trading convention is to quote FRN prices in terms of a discount spread (Market spread). *The discount margin is defined as the spread above the index that equates the price of the FRN to the present value of the projected cash flows*.

To calculate the price of the FRN the following information is required.

- Issuance spread ("IS"), for example 45bps above 3M Jibar
- Market spread ("MS)
- The last reset rate. This is 3m Jibar rate set on the last coupon date
- The settlement date
- Last Reset Date
- Maturity Date
- Mid Zero swap rate at each coupon date
- Forward rate at each coupon date

The tables below are an example, using the discount margin method to price a 1 year FRN on issue date.

Input table

Notional	R 1,000,000.00
Settlement Date	15/10/2012
Maturity Date	15/10/2013
Last Reset Date	15/10/2012
Reset Rate	5.0750%
Issue Spread	45bp
Market spread	45bp
Day count	365

Value date	15/10/2012
Accrued Interest	-
ALL IN PRICE	1,000,000.00

Start date	End date	Day Count	Swap Zero	Discount factor	Forward rate	Interest	Total payments	Period DF	Zero DF	PV CF
15/10/2012	15/01/2013	0.25205	4.7600%	1.00000000	5.0750%	13,926.03	13,926.03	0.9862652	0.986265242	13,734.76
15/01/2013	15/04/2013	0.24658	5.0750%	0.98736978	4.8600%	13,093.15	13,093.15	0.9870761	0.973518813	12,746.43
15/04/2013	15/07/2013	0.24932	4.9994%	0.97567769	4.7700%	13,014.25	13,014.25	0.9871529	0.961011961	12,506.85
15/07/2013	15/10/2013	0.25205	4.9626%	0.96421097	4.7900%	13,207.67	1,013,207.67	0.9869645	0.948484689	961,011.96

3.1 Calculation of the forward rate at each coupon date

Once the mid swap zero rates have been derived for each coupon date, the forward rates are then derived. The forward rates are implied from the Zero Swap curve and can be calculated using the following formula:

$$(1 + t_{i-1}^* r_{i-1}) \cdot \left(1 + \tau_i \cdot r_i^{fwd}\right) = (1 + t_i^* r_i)$$

$$\left(1 + \tau_i \cdot r_i^{fwd}\right) = (1 + t_i^* r_i)/(1 + t_{i-1}^* r_{i-1})$$

$$\left(1 + \tau_i \cdot r_i^{fwd}\right) = \frac{df_{i-1}^*}{df_i^*}$$

$$\boxed{\tau_i \cdot r_i^{fwd} = \left(\frac{df_{i-1}^*}{df_i^*} - 1\right)}$$

Where: $\tau_i = (t_i^* - t_{i-1}^*)/365$ is the period between the coupon dates.

df_i is the discount function between coupon dates.

r_i^{fwd} is the forward rate at coupon date.

Now, using the above formula we can calculate the forward rates at date 15/10/2012 and 15/01/2013 respectively shown on the pricing table above.

Fwd rate = (1/0.9873698-1)*365/92
*   = (1.0127918-1)*3.9673913*
*   = 0.01279178155361050*3.9673913*
*   = 0.05075*

Fwd rate = (0.9873698/0.97567769-1)*365/90
*   = (1.0119836-1)*4.0555556*
*   = (0.01198361)*4.0555556*
*   = 0.0486*

3.2 Calculating the coupon amount

Example1 (@ coupon date 15/01/2013)

| **Coupon = Nominal x (FWD rate + Issue spread)* number of days /365** |

= 1 000 000 *(0.05075 + 0.0045)* 92/365

= 1 000 000*(0.05525)*0.25054795

= **13 926.03**

Example2 (@ coupon date 15/04/2013)

Coupon = Nominal x (FWD rate + Issue spread)* number of days /365

= 1 000 000 *(0.0486+ 0.0045)* 90/365

= 1 000 000*(0.0531)*0.246575342

= **13 093.15**

3.3 Calculating the present value (PV) of the coupon cash flows

| **DF (forward period)   =   1/ (1 + forward rate + market spread* ($period_i$ - $period_{i-1}$)/365)** |

@ date 15/01/2013   = 1/(1 + (0.05075+ 0.0045)*92/365)

= 1/ (1+ 0.05525*0252054795)
= 1/ (1+ 0.0139260273972603)
= 1/ 1.0139260273972603
= **0.9862652432**

@ date 15/04/2013 = 1/(1+ (0.0486 + 0.0045)*90/365)
= 1/ (1+0.0531*0.2465753)
= 1/ (1+0.0130932)
= 1/ (1.0130932)
= **0.9870761**

Therefore the Zero DF　= 0.9862652432*0.9870761
　　　　　　　　　　= **0.973518813**

PV at each coupon date is calculated as follows:

PV = Zero DF x Coupon

= 0.9862652432*13 926.0.3
= 13 734.76

PV = Zero DF x Coupon

= 0.973518813 * 13 093.12
= 12 746.43

Then sum of all the present values at each coupon date is the price of the FRN.

The sum is represented mathematically by the formula below:

$$PV_{FRN} = \sum_{i=1}^{n}\left(N.\tau_i.r_i^{fwd}.df_i\right) + N.df_n$$

$$PV_{FRN} = N.\tau_1.r_1^{reset}.df_1 + N.\tau_2.r_2^{fwd}.df_2 + \cdots + N.\tau_n.r_n^{fwd}.df_n + N.df_n$$

Where:

PV_{FRN} = price of the FRN

N = nominal

$\tau_i = (t_i - t_{i-1})/365$ is the period between the coupon dates.

r_i^{fwd} = is the forward rate at the coupon dates.

df_i = is the zero discount function at each coupon date which is the **product** of the forward period discount function.

The following rule applies to FRNs:

- Trades at PAR, if IS = MS, the issue and the reset dates are the same.
- Trades at a DISCOUNT, if MS > IS
- Trades at a PREMUIM, if MS < IS

Example1 of a buyback FRN, where Market Spread (MS) is less than the Issue Spread(IS)

Notional	R 1,000,000.00
Value date	13/11/2012
Settlement Date	13/11/2012
Maturity Date	04/04/2014
Days to Maturity	507.00
Prev Reset Date	05/11/2012
PrevReset Rate	5.07500%
Reset Frequency	3m
Issue Spread	90bp
Market spread	30bp

Accrued Interest	1,309.59
CLEAN PRICE	1,007,977.77
ALL IN PRICE	1,009,287.36

Start date	End date	Term	Fwd Start date	Fwd End date	Rate at start	Rate at end	Forward rate (simp)	Interest	Notional	Total payments	Period DF	Zero DF	PV CF
05/11/2012	04/01/2013	0.164	05/11/2012	04/01/2013		0.0503		9,821.92	-	9,821.92	0.992401	0.992401	9,747.28
04/01/2013	04/04/2013	0.247	04/01/2013	04/04/2013	0.0503	0.0503	0.0500	14,548.69	-	14,548.69	0.987099	0.979598	14,251.87
04/04/2013	04/07/2013	0.249	04/04/2013	04/07/2013	0.0503	0.0499	0.0489	14,442.44	-	14,442.44	0.987219	0.967078	13,966.96
04/07/2013	04/10/2013	0.252	04/07/2013	04/10/2013	0.0499	0.0497	0.0490	14,629.14	-	14,629.14	0.987053	0.954557	13,964.35
04/10/2013	06/01/2014	0.258	04/10/2013	06/01/2014	0.0497	0.0499	0.0501	15,220.29	-	15,220.29	0.986509	0.941679	14,332.64
06/01/2014	04/04/2014	0.241	06/01/2014	04/04/2014	0.0499	0.0501	0.0506	14,377.45	1,000,000.00	1,014,377.45	0.987234	0.929658	943,024.26

Example2 of a buyback FRN, where Market Spread(MS) is greater than the Issue Spread(IS)

Notional	R 1,000,000.00
Value date	13/11/2012
Settlement Date	13/11/2012
Maturity Date	04/04/2014
Days to Maturity	507.00
Prev Reset Date	05/11/2012
PrevReset Rate	5.07500%
Reset Frequency	3m
Issue Spread	90bp
Market spread	118bp

Accrued Interest	1,309.59
CLEAN PRICE	996,286.79
ALL IN PRICE	997,596.38

Start date	End date	Term	Fwd Start date	Fwd End date	Rate at start	Rate at end	Forward rate (simp)	Interest	Notional	Total payments	Period DF	Zero DF	PV CF
05/11/2012	04/01/2013	0.164	05/11/2012	04/01/2013		0.0503		9,821.92	-	9,821.92	0.991167	0.991167	9,735.17
04/01/2013	04/04/2013	0.247	04/01/2013	04/04/2013	0.0503	0.0503	0.0500	14,548.69	-	14,548.69	0.984990	0.976290	14,203.74
04/04/2013	04/07/2013	0.249	04/04/2013	04/07/2013	0.0503	0.0499	0.0489	14,442.44	-	14,442.44	0.985085	0.961729	13,889.70
04/07/2013	04/10/2013	0.252	04/07/2013	04/10/2013	0.0499	0.0497	0.0490	14,629.14	-	14,629.14	0.984897	0.947203	13,856.77
04/10/2013	06/01/2014	0.258	04/10/2013	06/01/2014	0.0497	0.0499	0.0501	15,220.29	-	15,220.29	0.984309	0.932341	14,190.49
06/01/2014	04/04/2014	0.241	06/01/2014	04/04/2014	0.0499	0.0501	0.0506	14,377.45	1,000,000.00	1,014,377.45	0.985171	0.918515	931,720.50