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BESA FLOATING RATE NOTE (FRN) PRICING SPECIFICATION

This document provides the pricing methodology used by BESA and endorsed by the Quantitative Committee for the pricing and valuation of Floating Rate Notes.

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1 INTRODUCTION

There are many different Floating Rate securities, or simply *floaters,* which all have a common feature: coupon interest will vary over the instrument's life. Although a floater's coupon can depend on a wide array of economic variables (FOREX rates, commodities etc), a floater's coupon payments usually depend on the level of money market interest rates (short term interest rates e.g. JIBAR, T-Bills, Prime etc). Reset of the coupon rate is usually quarterly in South Africa, however, semi- annually and monthly resets also exist.

The coupon formula for a floater is expressed as:

Coupon Rate = reference rate ± quoted spread

Each instrument will get issued with a certain spread over the benchmark rate (depending on the issuer's credit rating, term to maturity etc). The most popular reference rate in South Africa is the 3 month JIBAR (Johannesburg Inter-Bank Agreed Rate). The JIBAR rate is based on the average of the NCD rates quoted by the large banks in South Africa after eliminating \ dropping off the highest and lowest outliers.

The fundamental benefit of floating Rate Notes is that the security will pay coupons based on the prevailing interest rates. At any one time we will only know what the value of the next coupon \ cash flow will be, since we don't know what the actual 3 month rates will be in years to come! As a result, the pricing of these instruments is usually considered to be slightly more complicated than that of Vanilla instruments.

These instruments are typically traded on price (All-in Price) per R100 nominal.

2 SUMMARY

Notwithstanding that these instruments are being priced based on All-in price per R100 nominal, the valuation generated by BESA is based on receiving quotes from market participants with respect to the change in credit spread using the BEASSA perfect fit SWAP zero curve or the BESA Linear Zero Swap Curve as the reference curve. Once the credit spread has been received from the market participants, the following methodology is used to determine the All-in price of the FRN:

- Generate a schedule of coupon payment dates.
- Determine the Zero SWAP rate corresponding to the coupon date.
- Calculate the forward rate for each prospective coupon date
- Calculate the implied future value of the prospective coupons i.e. predict the future coupons using the forward curve.
- Discount each coupon and principal back to the present, taking into account the market's perception of what the new trading spread should be.

3 METHODOLOGY FOR CALCULATING THE ALL-IN PRICE FOR THE FLOATING RATE NOTE

FRN's are priced based on All-in price per R100 nominal. The pricing methodology outlined in this document summarises the generic methodology of how FRN's are priced on BESA for valuation purposes (mark-to-market). The bond capture system (BTB) does not use this methodology for pricing FRN's and will only accept a price as an input.

Traditionally when an issuer issues these instruments they would decide on a relatively liquid index to set their coupons against. The norm in South Africa with respect to Corporate FRN's is either the JIBAR rate (most popular) or the prime rate or as in the case of the Government FRN's, the Treasury Bill rate. Since the JIBAR rate is the mid of the NCD's (Negotiable Certificates of Deposit), quoted by the large banks in South Africa, it is effectively based on the credit quality of these banks. Likewise, the Treasury Bill rate is based on the credit quality of South African government. As a result, since many organisations who issue these instruments do not have a credit rating as good as the spread implied in JIBAR with that of JIBAR, issuers often pay a spread over the JIBAR rate or Treasury Bill rate.

This spread (expressed in basis points) will generally be the interest rate required by the market \ investors in addition to that of the index, such that the issuer will be able to issue at par or close thereto, given the credit quality of issuer at the time of issuance. It is important to note that this spread is based on the market's perception of the credit quality at the time of issuance. Although the market perception of what this credit spread should be for the issuer will change over time, the coupon payments are made based on what was set at the time of issuance!

The methodology described in this document assumes quarterly payments. In addition, we assume that cash is paid on coupon dates, unless the coupon date is a Saturday, Sunday or public holiday, in which case the cash is paid on the next business day and the coupon payment incorporates these extra days. However, one must consult the terms and conditions in the issuers placing document for more details as there are some issues which do not adopt this convention. For example if the payment date falls on a Saturday, some issuers will pay based on the next business day (irrespective of whether it falls into a new month or not i.e. we do not use modified following) and include the extra days of interest in the payment, whereas other issuers pay interest based on the next business day principle but won't pay the interest based on the extra days. Instead, these days are incorporated in the new period.

Since the majority of FRN's listed are linked to JIBAR, the Zero SWAP curve is used. In particular, the BEASSA perfect fit Zero SWAP curve or the BESA Linear SWAP Curve. It is from these curves that we derive what the market believes the future JIBAR rates are. These forward rates are implied from the "Spot" rates *(strictly speaking, the term Zero coupon rates is reserved for those rates observed in the market versus "spot rates" which are derived from a process called bootstrapping.)* We then use these rates (plus the spread at issuance) to calculate the prospective coupons as well as discount the prospective coupons and principal back to the present.

In addition, since the market perception of the credit spread to JIBAR is likely to be different to the spread when the issuer originally issued the bond, we would also have to discount the change in spread back to the present.

The following inputs need to be **known** in order to price a FRN:

- Initial spread ("IS) – this is the margin at which the FRN is issued. For example, a FRN that bears interest at JIBAR + 50bp would have an IS of 50bp.
- The last interest reset rate. This is given to us based on the quarterly rate set on the last coupon date.
- The number of days in the current interest period, i.e. the number of days from the last coupon date to the next coupon date.
- The number of days from settlement date until the next coupon date.
- The payment frequency of the FRN (e.g. semi-annual, quarterly etc)
- The values from a Zero SWAP Curve corresponding to the prospective coupon dates.
- The Trading spread –this can be seen as what the initial spread would be if the FRN was issued today. The initial spread changes due to numerous criteria: e.g. changing credit characteristics of the issuer (and or sector), the prevailing interest rate environment, etc.

On each coupon payment date, the next coupon is reset. For instance, for a quarterly paying floater, we will only know what the second quarter payment is on the day of payment of the first quarter coupon.

3.1 Determine the Coupon Payment Dates

The Coupon payment dates are set out in the placing document for the listed bond. However, these dates may not coincide exactly with the actual payments dates for any particular year due to holidays or weekends. The general convention in the market is that the coupon will be paid on the next business day. The bondholder will be compensated for the extra days interest as it is included in the day count. *(Please note that there are exceptions, however for BESA valuations this convention will always be assumed.)*

3.2 Determine the Zero SWAP Rate corresponding to each coupon date.

BESA currently publishes three Zero SWAP curves, the BEASSA Perfect fit SWAP curve, the BEASSA best decency curve and the BESA Linear Interpolation curve. The two curves that will be used for the valuation of the FRN's are the BEASSA Perfect Fit curve and the BESA Linear curve. Please note that the use of these two curves will produce different results as they derive the curve from different methodologies. The choice between which curve is more appropriate is personal. BESA will value the FRN's using the both methodologies independently. For more information, please refer to the BESA website www.bondexchange.co.za

3.3 Treatment of First Coupon

A floating rate note is not always issued such that the issue date corresponds to one on the coupon payment dates (only the day & month are considered). A stub period or broken first coupon is referred to when the month and day of issue does not correspond to the month and day of any coupon payment dates i.e. the time to first coupon payment is long or shorter than the remainder or the coupon payments. As a result of the broken or stub period, the treatment of the first coupon might be different from issue to issue. This period may be a long or short stub. A short stub refers to the payment of the first coupon on the 1st sequential coupon payment date after the issue date. A long stub refers to the 2nd sequential. The date of the first coupon payment date, either long or short stub will be indicated in the calculation supplement of the placement document

When the next coupon date is effectively the1st coupon payment date i.e. settlement date < 1st interest payment date the coupon calculation is as follows:

$$C@NCD = 100 \times (Y_{issue}) \frac{d}{365}$$

And

$$C_{1^{st}cd} = C_{ncd} = b \times C@NCD$$

Where:

$C@NCD$ = The coupon amount to be paid at the next coupon date, expressed as a Rand amount per R100 nominal.

$C_{1^{st}cd}$ = The effective coupon paid on the 1^{st} coupon payment date (NCD)

b = Cum / Ex Indicator (1 if the FRN is cum, 0 if it is ex). A FRN is *cum* if settlement date is before books close date and *ex* if it is on or after the books closed date.

Y_{Issue} = First coupon rate (sourced from the pricing supplement. This rate usually include an initial spread)

d = Number of days in current interest period. For a full coupon stub

$$d = 1^{st}\ \text{IPD} - \text{ID}$$

where:

1^{st} *IPD* = 1^{st} Interest Payment Date (Specified in the Placement document)

ID = Issue Date or 1^{st} Settlement Date (Specified in the Placement document)

3.4 Calculate the forward rate for each coupon date

Once the spot rates have been derived for each coupon payment date, the forward rates then need to be derived. These forward rates are implied in the Zero SWAP curve and can be derived from the following formula:

$$FR_t = \frac{365}{D_t - D_{t-1}}\left[\frac{\left(1+\frac{Z_t}{4}\right)^{\frac{4\times(D_t - Sd)}{365}}}{\left(1+\frac{Z_{t-1}}{4}\right)^{\frac{4\times(D_{t-1} - Sd)}{365}}} - 1\right]$$

Equation 1

Where:

t is an integer and refers to the respective coupon payment dates and applies from the coupon date following the next coupon payment date to maturity (n) (t-*1 is the coupon payment date preceding t*).

FR_t = Forward rate applicable to period t.

D_t = The date (using the Modified Following convention) for coupon date t.

Sd = Settlement Date

Z_t = the zero rate for the tth period, expressed as NACQ taken from the BEASSA perfect fit Zero SWAP curve or the BESA Zero Linear SWAP curve.

3.4.1 Last Coupon Payment (LCD) between Trade and Settlement

Coupons are reset quarterly when the trade date is equal a coupon payment day. The coupon reset is generally function of 3 month JIBAR + Spread. When however the last coupon date (LCD) falls between the trade and settlement date, the value of next coupon payment would not have been set. We therefore estimate JIBAR for the next coupon payment as we do for the remaining coupons. JIBAR for the next coupon payment is estimated using *Equation 1* with the following substitutes:

t = Next Coupon Date (NCD)
t – 1 = Last Coupon Date (LCD)
Sd = Trade Date

3.5 Calculate the next coupon and predict the future value of each Coupon (plus spread).

The next coupon payment is treated differently from the other prospective coupons, as we will always know what the next coupon is. Therefore:

$$C@NCD = 100 \times \left(JIBAR_{Fix} + IS\right)\frac{d}{365}$$

$$C_{ncd} = b \times C@NCD$$

Where:

C_{ncd} = The next coupon to be paid, expressed as a Rand amount per R100 nominal.

b = Cum / Ex Indicator (1 if the FRN is cum, 0 if it is ex). A FRN is *cum* if settlement date is before books close date and *ex* if it is on or after the books closed date.

$JIBAR_{Fix}$ = The floating benchmark rate from last reset date to next coupon payment date. Please note that this rate is expressed on a simple basis. If there is a broken first coupon this rate, first coupon rate used is to be sourced from the pricing supplement.

IS = The Initial spread, which is set on Issue Date (spread over Floating benchmark rate)

d = Number of days in current interest period

d = NCD – LCD

where:

LCD = Last Coupon Date

NCD = Next Coupon Date

3.5.2 Last Coupon Date (LCD) between Trade and Settlement

Should the last coupon date (LCD) fall between a trade a settlement, the next coupon would not have been fixed. As a result, we use the Zero SWAP curve to derive the forward rate

$$C@NCD = 100 \times \left(FR_t + IS\right)\frac{d}{365}$$

$$C_{ncd} = b \times C@NCD$$

Where :

FR_t = The forward rate discussed in section 3.3.1 applicable to period t.

3.5.3 Predict Future Coupons

Since the issuer only determines the next coupon to be paid on the coupon payment date, the balance of the prospective coupons are not known. As a result, we use the Zero SWAP curve to derive the implied forward starting JIBAR rate as per section 3.3.

$$C_t = R100 \times \left(\frac{(FR_t + IS) \times (D_t - D_{t-1})}{365} \right)$$

Where:

t is and integer referring to the whole coupon payment period from t=2 to n (maturity).

C_t = The predicted coupon applicable to period t expressed per R100

FR_t = The forward rate as previously calculated in section 3.3 applicable to period t.

IS = The initial spread as determined at issue Date (spread over Floating benchmark rate).

3.6 Discount each coupon and principal back to the present

Now that we have determined the next coupon and the balance of the prospective coupons we can discount these coupons and principal back to the present using the zero Swap rates corresponding to the coupon payments and principal payments. The rate at which we discount these coupons is the zero Swap rate (expressed as NACQ) and the trading spread. Please note that the spread that is used to discount the coupons and principal is the traded spread (TS), which is not necessarily the same as the initial spread (IS). This trading spread represents the markets perception of what the credit spread of the issuer should be at present.

$$AIP_{FRN} = \sum_{t}^{n} \left(\frac{C_t}{\left(1 + \frac{(Z_t + TS)}{4}\right)^{\frac{4 \times (D_t - Sd)}{365}}} \right) + \frac{R100}{\left(1 + \frac{(Z_n + TS)}{4}\right)^{\frac{4 \times (D_n - Sd)}{365}}}$$

where:

AIP_{FRN} = The All-in price of the FRN expressed per R100 nominal.

TS= Trading Spread over the floating benchmark

D_t = The coupon date (using the Modified Following convention) corresponding to time t where t runs from the next coupon date to maturity (n); t includes the next coupon date).

C_t = The predicted coupon applicable to period t expressed per R100 including C_{ncd} which has a value other than 0 in the cum period

Sd = Settlement Date

Z_t = the zero rate for period t, expressed as NACQ taken from the BEASSA perfect fit Zero SWAP curve or the BESA Zero Linear SWAP curve.

3.7 Accrued Interest

$$AI_{frn} = \frac{365}{d} \times C\,@\,NCD \times \left(\frac{b \times d - w}{365} \right)$$ therefore:

$$AI_{frn} = C\,@\,NCD \times \left(\frac{b \times d - w}{d} \right)$$

Where:

$C\,@\,NCD$ = The next coupon to be paid, expressed as a Rand amount per R100 nominal.

b = Cum / Ex Indicator (1 if the FRN is cum, 0 if it is ex). An FRN is *cum* if settlement date is before books close date and *ex* if it is on or after the books closed date.

d = Number of days in current interest period

If S < 1st Coupon payment date:

$$d = 1^{st}IPD - ID$$

else

$$d = NCD - LCD$$

where:

LCD = Last Coupon Date

NCD = Next Coupon Date

1^st IPD = 1st Interest Payment Date

ID = Issue Date

If S < 1st Coupon payment date :

w = *1st IPD* – Settlement date

else

w = NCD – Settlement date

3.8 An intuitive look at the Pricing of FRN's

There are a few common misperceptions about FRN's namely:

- The price of the FRN can never go below R100.
- The price of the FRN will always be R100 on coupon payment date.

The purpose of this section is to dispel these misconceptions by simplifying the pricing formula and re-arranging it to make it easier to understand. The simplifications are:

- Assume coupon payments are annual.
- Assume the valuation date equals the reset date.
- The coupon rate (this time expressed as a percentage) C_t is made up of $C_t = FR_t + IS$ where FR_t is the forward interest rate derived from the Zero Swap curve and IS is the initial spread.

We know that the pricing of a FRN is the process of discounting the next coupon due back to present (for which we know the exact cash value) as well predicting the future coupons implied by the Zero SWAP curve and discounting these predicted coupons and the principal back to present.

The simplified All-in price of the FRN can be given by:

$$AIP_{FRN} = 100 \times \left[\sum_{t}^{n} \frac{FR_t + IS}{(1 + Z_t + TS)^t} + \frac{1}{(1 + Z_n + TS)^n} \right]$$

Where:

Z_t = The JIBAR rate applicable to period t, expressed NACA. Note, since we assumed that the valuation date is a coupon date, Z_t will therefore equal the next reset rate for the next coupon date i.e. $Z_t = FR_t$ for the next coupon date.

FR_t = The derived forward rate from the zero Swap curve for period t.

Since $(1 + Z_2 + TS)^2 = (1 + Z_1 + TS)(1 + FR_2 + TS)$ i.e. the zero spot rate of year 2 is a function of the value of the spot rate in year 1 and the forward rate starting at the end of year 1 for the period of year 2 (FR_2). Therefore we can represent the term of the second discounted "predicted coupon" as:

$$\frac{FR_2 + IS}{(1 + Z_2 + TS)^2} = \frac{FR_2 + IS}{(1 + Z_1 + TS)(1 + FR_2 + TS)}$$

and year 3 would be represented by:

$$\frac{FR_3 + IS}{(1 + Z_3 + TS)^3} = \frac{FR_3 + IS}{(1 + Z_1 + TS)(1 + Z_2 + TS)(1 + FR_3 + TS)}$$ and so on.

Therefore all future coupons and principal can be discounted by:

$$AIP_{FRN} = 100 \times \left[\sum_{t}^{n} \frac{FR_t + IS}{(1 + Z_t + TS) \ldots (1 + FR_n + TS)} + \frac{1}{(1 + Z_t + TS) \ldots (1 + FR_n + TS)} \right]$$

Now to confuse things, add (TS-TS) to the equations

$$AIP_{FRN} = 100 \times \left[\sum_{t}^{n} \frac{FR_t + IS + (TS - TS)}{(1 + Z_t + TS) \ldots (1 + FR_n + TS)} + \frac{1 + (TS - TS)}{(1 + Z_t + TS) \ldots (1 + FR_n + TS)} \right]$$

$$AIP_{FRN} = 100 \times \left[\left[\sum_t^n \frac{FR_t + TS}{(1+Z_t+TS)...(1+FR_n+TS)} + \frac{1}{(1+Z_t+TS)...(1+FR_n+TS)}\right] + \left[\sum_t^n \frac{IS-TS}{(1+Z_t+TS)...(1+FR_n+TS)}\right]\right]$$

Although this initially appears to be rather confusing, it displays some valuable information with respect to FRN's. At the beginning of each reset period (i.e. when the valuation date equals the reset date), the first term in brackets is an instrument trading at par because $Z_t = FR_t$ and the second term in brackets is an annuity paying the difference between the initial spread and trading spread This can be summarised as:

$$AIP_{FRN} = 100 + PV_{annuity}(IS - TS)$$

Therefore, if the IS=TS then the price of the FRN is Par on reset date. For any date up to the reset date, it is the present value of the next coupon, using the JIBAR applicable to the time remaining to the next coupon plus the trading spread as the discount rate.

When TS>IS, the FRN will be trading at a discount and when TS<IS, the FRN will be trading at a premium. This formula dispels some common misconceptions with respect to FRN's namely:

- The price of the FRN can never go below R100. This is not the case if the Trading spread is greater than the initial spread. In addition, even if the TS=IS the price of the bond will still "dip" below 100 in the ex period!
- The price of the FRN will always be R100 on coupon payment date. Again this will only be the case if TS=IS.

4 Duration

Please note that BESA will not be publishing the duration or delta of the FRN. This section has been included for interest purposes only.

Academically, the most common way at looking at the duration of a FRN is to treat it as an instrument with a short maturity i.e. the next reset date. Just before the reset date, we know that the coupon will be set to the prevailing interest rate. The FRN is then similar to cash, or a money market instrument, which has no interest rate risk (or at least very little) and hence is selling at par or zero duration.
Just after the reset date, the investor is locked into a fixed coupon over the accrual period (3 months). The FRN is then economically equivalent to a zero coupon bond with maturity equal to time to the next reset date. This kind of analysis is however only correct if the trading spread is equal to the initial spread.

Let's have a look at the duration of a FRN on a simplistic basis mathematically. Using the analysis from 3.7 above:

$$AIP_{FRN} = 100 \times \left[\left[\sum_{t}^{n} \frac{FR_t + TS}{(1+Z_t+TS)\ldots(1+FR_n+TS)} + \frac{1}{(1+Z_t+TS)\ldots(1+FR_n+TS)}\right] + \left[\sum_{t}^{n} \frac{IS-TS}{(1+Z_t+TS)\ldots(1+FR_n+TS)}\right]\right]$$

Duration of the FRN can be viewed as a function of Forward curve, keeping the trading spread constant as well as the trading spread, keeping the Forward curve constant.

4.1 Forward Curve Duration

In order to simply matters, initially let us assume that the IS=TS and then shift the Forward curve (derived from the Zero Swap Curve as before) on a parallel basis. This is represented as:

$$NewAIP_{FRN} = 100 \times \left[\left[\sum_{t}^{n} \frac{FR_t + \Delta i + TS}{(1+Z_t+\Delta i+TS)\ldots(1+FR_n+\Delta i+Ts)} + \frac{1}{(1+Z_t+\Delta i+TS)\ldots(1+FR_n+\Delta i+TS)}\right] + \left[\sum_{t}^{n} \frac{IS-TS}{(1+Z_t+\Delta i+TS)\ldots(1+FR_n+\Delta i+TS)}\right]\right]$$

$$NewAIP_{FRN} = 100 \times \left[\sum_{t}^{n} \frac{FR_t + \Delta i + TS}{(1+Z_t+\Delta i+TS)\ldots(1+FR_n+\Delta i+TS)} + \frac{1}{(1+Z_t+\Delta i+TS)\ldots(1+FR_n+\Delta i+TS)}\right]$$

Since all of the projected cash flows equal par after the next coupon date, the duration of an FRN is the same as duration of a zero coupon bond which matures at the next reset date when TS=IS.

What if the Trading spread is different from that of the initial spread? Using the equation above again, it is interesting to note that if the Trading spread is significantly greater than that of the initial spread, this could make the modified duration of the FRN become negative.

$$AIP_{FRN} = 100 \times \left[\left[\sum_{t}^{n} \frac{FR_t + TS}{(1 + Z_t + TS)\ldots(1 + FR_n + TS)} + \frac{1}{(1 + Z_t + TS)\ldots(1 + FR_n + TS)} \right] + \left[\sum_{t}^{n} \frac{IS - TS}{(1 + Z_t + TS)\ldots(1 + FR_n + TS)} \right] \right]$$

From the equation above, we can view the bond as a portfolio of a long position in a par floater and a long position in an annuity paying IS-TS (or in the event TS>IS, it would be considered as a short position in an annuity paying TS-IS). We know that the long position in the par portion of the "portfolio" will have a duration of a zero coupon bond with its maturity as the next coupon payment date; whereas, the annuity portion will reduce the duration if the FRN is trading at a discount and will increase the "portfolio" duration if the FRN is trading at a premium.[1]

4.2 Spread Duration

We define the spread duration as the price sensitivity of an FRN with respect to changes in the trading spread.

Going back to equation in section 3.7,

$$AIP_{FRN} = 100 \times \left[\sum_{t}^{n} \frac{FR_t + IS}{(1 + Z_t + TS)^t} + \frac{1}{(1 + Z_n + TS)^n} \right]$$

it is clear that an increase in trading spread (TS) affects the denominator of the equation but is not accompanied by an increase in the numerator. In fact, increasing the TS on a FRN will have the same price effect as increasing the yield on a fixed coupon bond which pays the projected coupons out to the maturity of the floater. This measure will be affected by the method used to predict the coupons[1].

[1] "Fixed Income Solutions – an Investor's Guide to Floating Rate Notes: Conventions, Mathematics, and Relative valuation", Raymond J.Iwanowski, (edited by Thomas S.Y.Ho), Irwin 1996

5 References:

"The Handbook of Fixed Income Securities", Frank J. Fabozzi, Mc Graw Hill, 6th Edition.

"Financial Risk Manager Handbook 2001 -2002", Philippe Jorion, Wiley Finance, 2001.

Fixed Income Solutions – an Investor's Guide to Floating Rate Notes: Conventions, Mathematics, and Relative valuation", Raymond J.Iwanowski, (edited by Thomas S.Y.Ho), Irwin 1996

6 Updates

Version 4.4 **10 November 2006**

Introduce new variable C@NCD detailing the Rand value of the next coupon. This was due to the Cum/Ex indicator effect of incorrectly assigning $C_{ncd} = 0$ when calculating the accrued interest.

- Shiresh Ranchhod

Version 4.3 **5 November 2006**

Revised broken first coupon to reflect full / partial 1st coupon.

Updates: * Accrued interest (3.7)

* Calculate the next coupon and predict the future value of each Coupon (plus spread). (3.5)

New: Treatment of First Coupon (3.3)

- Shiresh Ranchhod

Version 4.2 **09 October 2006**

Treatment of Implied Forward Coupon @ NCD

- Shiresh Ranchhod

Version 4.1 **03 October 2006-10-03**

Reflect a broken first coupon.

- Shiresh Ranchhod