CADIZ SECURITIES

ALWAYS. DERIVATIVE RESEARCH

THE NEW IMPLIED VOLATILITY INDEX WITH APPLICATIONS

THE SAFEX INTERBANK VOLATILITY INDEX (SIVX)

Grant Shannon and Manoshan Pillay

NOVEMBER 2006

cadiz FSG

ALWAYS. AHEAD

Financial Strategy Group

ALWAYS.

CADIZ DERIVATIVE RESEARCH

PROFESSOR DAVE BRADFIELD
Tel: 27 21 657 8351
e-mail: david.bradfield@cadiz.co.za

PETRUS BOSMAN
Tel: 27 21 657 8501
e-mail: petrus.bosman@cadiz.co.za

DR BRETT DUGMORE
Tel: 27 21 657 8552
e-mail: brett.dugmore@cadiz.co.za

YASHIN GOPI
Tel: 27 11 853 8006
e-mail: yashin.gopi@cadiz.co.za

DR JACO MARITZ
Tel: 27 21 657 8356
e-mail: jaco.maritz@cadiz.co.za

BRIAN MUNRO
Tel: 27 21 657 8352
e-mail: brian.munro@cadiz.co.za

MANOSHAN PILLAY
Tel: 27 21 657 8423
e-mail: Manoshan.Pillay@cadiz.co.za

ANGELA RIRIE
Tel: 27 21 657 8550
e-mail: angela.ririe@cadiz.co.za

GRANT SHANNON
Tel: 27 21 657 8508
e-mail: grant.shannon@cadiz.co.za

THABO TEMBO
Tel: 27 21 657 8506
e-mail: thabo.tembo@cadiz.co.za

THE NEW IMPLIED VOLATILITY INDEX WITH APPLICATIONS

ANALYST. GRANT SHANNON

Tel: 27 21 657 8508

e-mail: grant.shannon@cadiz.co.za

ANALYST. MANOSHAN PILLAY

Tel: 27 21 657 8423

e-mail manoshan.pillay@cadiz.co.za

EXECUTIVE SUMMARY

This research demonstrates the construction and application of an implied volatility index to the South African Financial markets.

Implied volatility is a central theme in this report. It is a forecast of asset return uncertainty, over a specific period of time, and is implied from the price of an option. An implied volatility index therefore, is a more transparent representation of this expected uncertainty, as it would typically be published on a daily basis by a financial market exchange.

The intention is to have this index published daily by SAFEX (South African Futures Exchange) from the first quarter of 2007 onwards. This will require input from both SAFEX and several index option market makers (the inter-bank market) to ensure that the daily data used to calculate the index is consistent and valid. The proposed index is thus labelled the SIVX (Safex Inter-Bank Volatility IndeX).

The results and applications in this report are referenced against equivalent research by several authors on the world's leading implied volatility index, the VIX. In this way the consistency of the SIVX results are tested and validated.

The initial part of this report discusses the construction of the proposed index followed by its application to the financial markets. These applications can be summarised as follows:

- An emerging market "Fear Gauge"
- A timing tool
- A political/country risk measure
- An underlying "spot" instrument for a volatility future, the SIVXF

CONTENTS

1. MOTIVATION FOR THE RESEARCH

The most referenced implied volatility index in financial literature is the VIX, a Chicago Board of Options Exchange product, first implemented twenty years ago.

Whaley (2000) labels this volatility index as "The Investor Fear Gauge". His paper considers the construction and application of volatility products to the USA markets. Several authors have contributed to the literature on volatility Indices. Derman (1998), in particular, discusses the merits of "Investing in Volatility" and makes the distinction between trading in implied volatility products (such as the VIX future) and realised volatility products (such as a variance swap).

A natural extension of this work would be to consider the application of similar themes to financial markets outside of the USA (or Europe). The South African market in particular is characterised by emerging market volatility, an interesting alternative to existing volatility product platforms. This report develops implied volatility products to a South African emerging market setting.

A starting point in this process is to work through broad definitions of volatility and risk.

2. VOLATILITY AND RISK

In financial markets, a typical reference to risk relates to the measurement of the dispersion of returns for a given portfolio or underlying asset. The less dispersed they are the less risk is perceived to be associated with these asset returns. The second moment (or variance) of a return distribution measures this dispersion.

A more practical representation of this risk or dispersion measure is the standard deviation of returns (the square root of the variance). This standard deviation is referred to as the asset price's volatility and can be calculated in one of two ways:

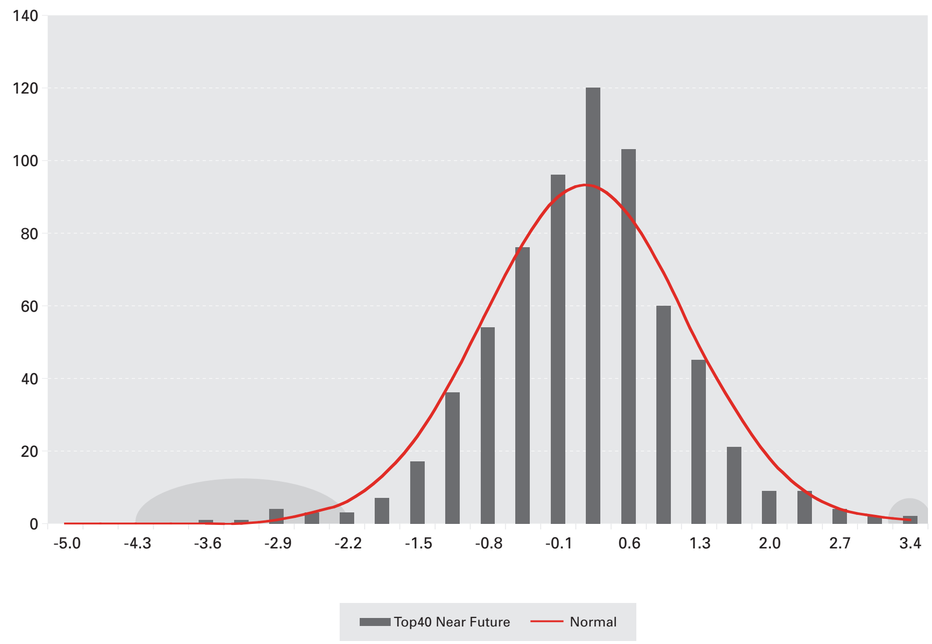
- From historical asset price return time series (see Appendix 1) in which case it is referred to as **historical** or **realised** volatility, or
- From option price time series data (using the Black Scholes option pricing model) in which case it is referred to as **implied** volatility.

The most important difference between an implied and realised volatility measure is that implied volatility is a forecast of asset return risk obtained from an option's price. It is an *expectation or forecast* of the markets perception of risk going forward.

Modelling this expectation is an important first step in the development of volatility products that can mitigate these risks. To achieve this, the market needs to be able to monitor this forecast on a daily basis in a transparent way. The construction of a volatility index enables this.

The question is, what underlying measure is more appropriate when building a volatility index – is it implied or historical volatility? Analysing the distribution of returns for a market index may provide some insight into the answer. Figure 1 plots the return distribution for two and a half years worth of daily TOP40 futures return data.

FIGURE 1: DISTRIBUTION OF RETURNS FOR TOP40 NEAR FUTURE

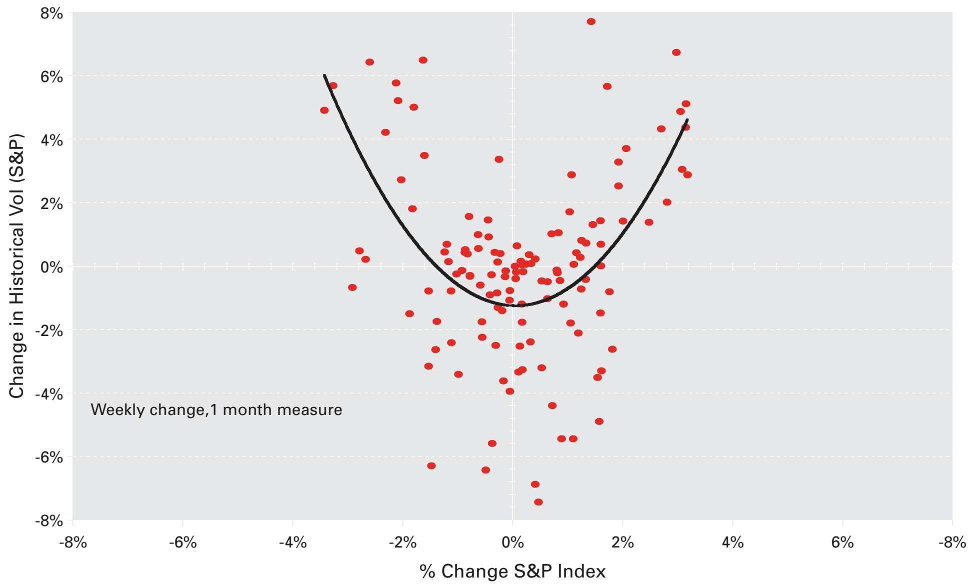


Notice how the TOP40 futures returns are more dispersed than that of the normal distribution, particularly on the left hand side of Figure 1. It seems reasonable therefore, to choose the volatility measure that most appropriately accounts for the non symmetric risk in the left and right hand side of this distribution.

Observing the change in historical or implied volatility relative to changes in the underlying market can expose the degree of symmetry associated with each volatility measure.

A reliable data set is needed to perform these initial tests of symmetry. Almost three years worth of weekly S&P implied volatility (VIX) data is compared with weekly S&P historical volatility data for the same period. Each measure of volatility refers to a one month time frame. Figures 2 and 3 illustrate the symmetry of the response of the two volatility measures to changes in the underlying market.

FIGURE 2: WEEKLY CHANGE IN HISTORICAL VOL VS. WEEKLY CHANGE IN S&P



From Figure 2 it is evident that historical volatility changes do not sufficiently differentiate risk in upward and downward markets i.e.: there is a symmetrical response to changes in historical volatility and changes in the market.

FIGURE 3: WEEKLY CHANGE IN VIX VS. WEEKLY CHANGE IN S&P

Implied volatility, in Figure 3, shows a non-symmetric response to changes in the market. This is an important feature of implied volatility – it illustrates how investors expect higher levels of risk in downward markets and expect lower levels of risk in upward markets. This is consistent with the non-symmetric risk return profile observed in Figure 1 earlier.

Implied volatility is thus chosen as the underlying measure for the construction of the proposed volatility index.

Note: There have been several attempts at listing volatility indices on various exchanges internationally. Those that have succeeded have been based on implied volatility. The discussions around Figures 2 and 3 help to explain why this is the case.

3. THE NEW VOLATILITY INDEX - INTRODUCTION

We choose to follow the original calculation methodology proposed by Whaley (2000). This approach is simpler and more intuitive than the reconstructed CBOE (2003) formula. Adjustments are made, however, to Whaley's method to accommodate the less liquid TOP40 option market i.e. only at- the-money (ATM) options are considered in the index construction process.

From a technical point of view the new implied volatility index will represent a daily measure of ATM volatility for a three month period. However, when compared with the current VIX formulation (CBOE, 2003) it represents a lower bound for three month volatility as a whole.

The current VIX formula comprises a strike based weighted *average of option prices* used to calculate the implied volatility index as opposed to the *average of volatilities* used in Whaley (2000). The VIX represents a forecast of one month return uncertainty for the S&P index using intra-day S&P option prices.

It is worth reiterating that the emerging market volatility index discussed in this report will reflect a measure of three month ATM volatility, on a daily basis. This is different to near ATM volatility which is typically a measure of three month implied volatility only once during its lifetime.

To calculate such a daily "rolling" three month measure of implied volatility will require representative figures for near and next-near ATM implied volatility, on a daily basis. The SAFEX mark-to-market dataset is used when ATM options trade actively on a given day. In the absence of sufficient trade volume, the suggestion is made that inter-bank market makers provide implied volatility "mid market" levels for the required options. These "market maker" levels will be averaged for each maturity, before being used to calculate the new index for the day concerned.

The proposed joint SAFEX and inter-bank contributions to the index valuation process hint at a possible name for the index i.e. The SIVX: "Safex Interbank Volatility IndeX".

We summarise the general specifications for the new SIVX as follows:

INDEX NAME:

SAFEX Inter-Bank Volatility Index (SIVX)

INDEX CONSTITUENTS

Average ATM implied volatility levels for near and next-near TOP40 index Puts and Calls.

INDEX VALUE

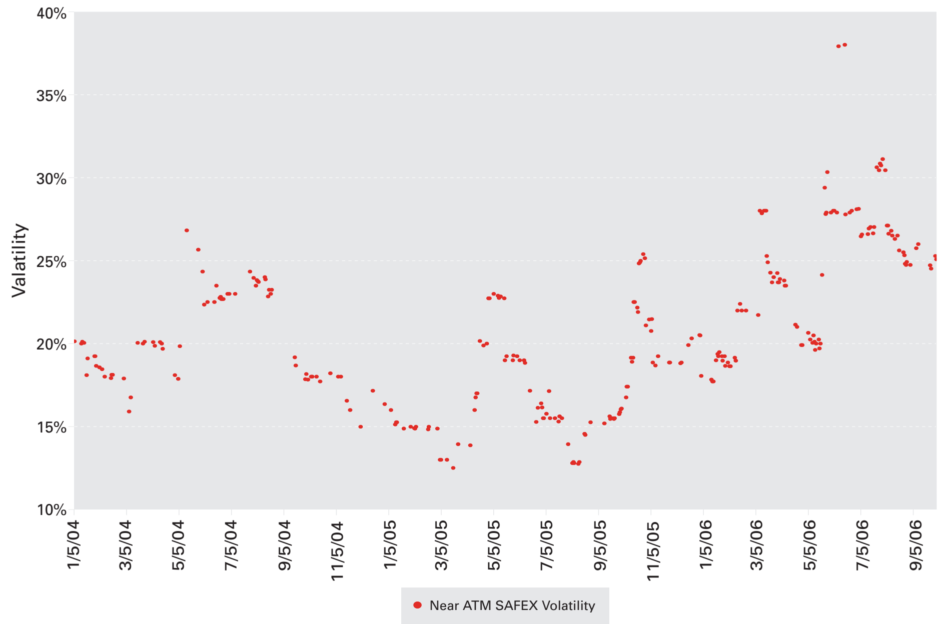
- The SIVX represents a daily ATM three month implied volatility measure – divide by 100 to get it's implied volatility percentage i.e: A SIVX value of 21.14 has an equivalent percentage representation of: 21.14% (0.2114) or a basis point representation of 2114 (0.2114 x 10,000).
- The index has a Rand value given that 1 basis point represents R10 (i.e. a multiplier of 10). Hence a SIVX value of 21.14, for example, will have a Rand value of R21,140 (0.2114x 10,000x10).

4. VOLATILITY INDEX CONSTRUCTION

To create a historical representation of the SIVX we use SAFEX implied volatility data (very limited historical inter-bank data is available, see Appendix 3). To minimise data inconsistency, SAFEX implied volatilities are obtained from options with active trade volume, on the day concerned. This reduced data set is further filtered according to the options delta (i.e. call options with deltas between 0.45 and 0.55 and put options with deltas between -0.45 and -0.55). These put and call implied volatilities are averaged to give an ATM or *close* to ATM measure for the expiry concerned.

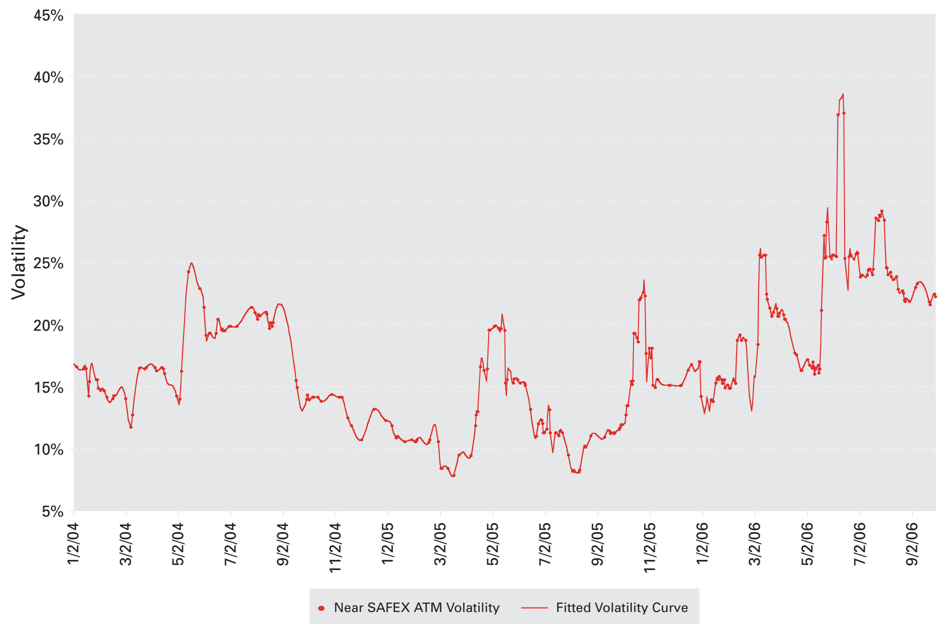
All SAFEX options between Jan 04 and Sep 06 are scanned in this way. The results provide an incomplete daily set of implied volatility data for the near and next-near contracts. Figure 4 illustrates this "sparse" daily evolution of near ATM volatility on the TOP40 options for the period concerned.

FIGURE 4: SPARSE DATA: ATM IMPLIED VOLATILITY (NEAR)



A curve is fitted to the data in Figure 4 to "fill the gaps". The estimated points approximate the inter-bank market levels on those days when no relevant SAFEX options traded. In practice, the gaps will be filled with the actual average levels provided by the inter-bank market. Figure 5 details the "re-constructed" daily evolution of near ATM volatility.

FIGURE 5: SMOOTH DATA: ATM IMPLIED VOLATILITY (NEAR)



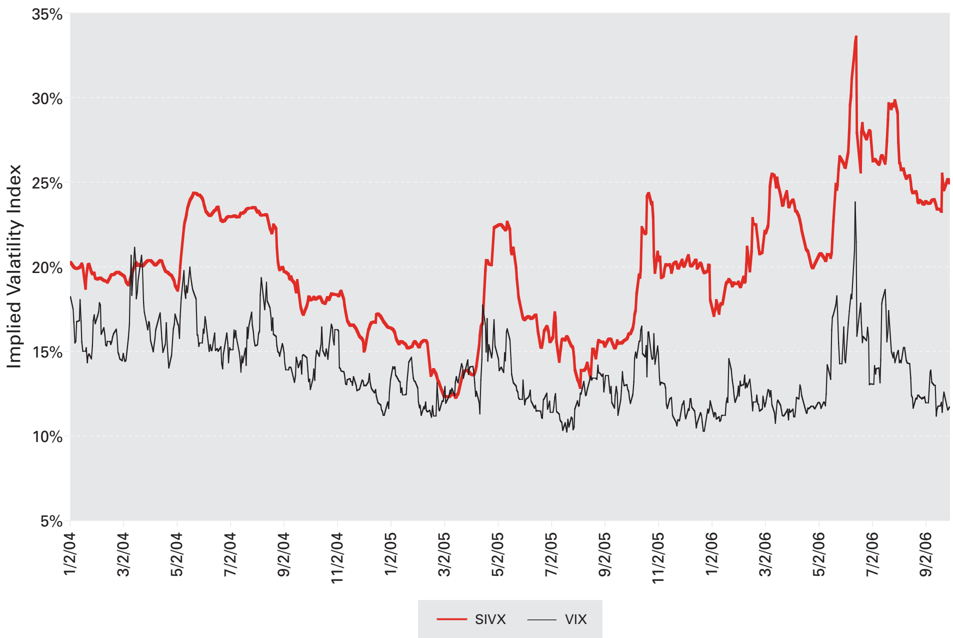
This procedure was repeated for the next-near contract. Approximately 50% of the data points had to be "fitted". Fortunately the sparse data points were sufficiently spread out to enable the applied mathematical algorithm to accurately fit the data.

Before considering the actual index construction it is worth noting that a test was performed to compare SAFEX implied volatilities against a small data base of inter-bank "broker" volatilities – for the exact same options on the same trading days. This was done to investigate the extent to which there may be irregularities between these two data sources. If so, the necessary adjustments need to be made to the data collection process going forward. The results are presented in Appendix 3.

5. THE SIVX - SAFEX INTERBANK VOLATILITY INDEX

The historical data is applied to the refined Whaley (2000) model to produce the SIVX. Figure 6 graphs the SIVX, over the test period Jan 04 to Sep 06.

FIGURE 6: THE SIVX (SAFEX INTERBANK VOLATILITY INDEX)

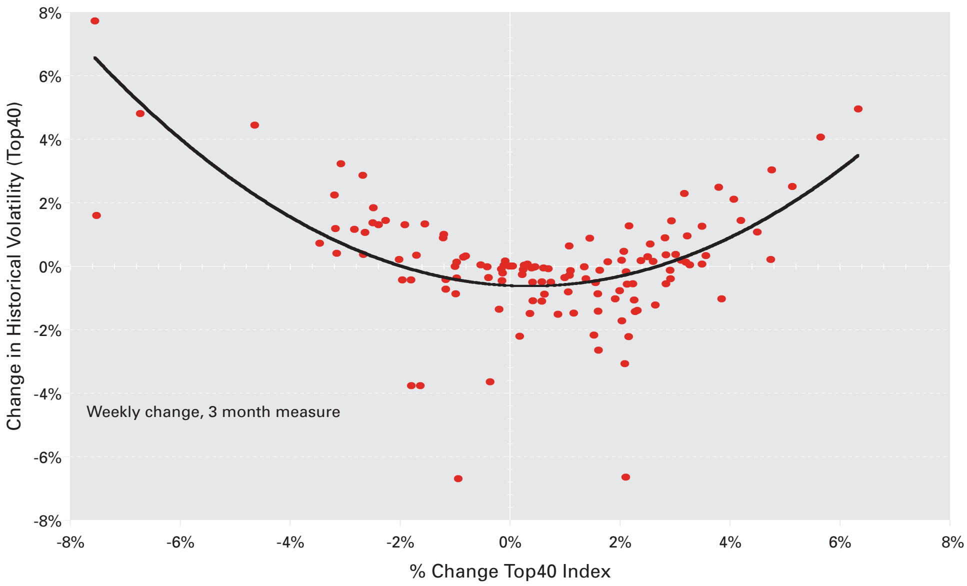


The red line represents the SIVX and the black line the VIX over the same period. The SIVX trades above the VIX – this makes sense intuitively as the South African markets are higher risk than that of the USA.

Figures 2 and 3 discussed earlier illustrated differences between implied and historical volatility risk perception. We apply the same tests to the SIVX to see if its behaviour is consistent with this.

Almost three years worth of weekly TOP40 implied volatility (SIVX) data is compared with weekly TOP40 historical volatility data for the same period. Each measure of volatility refers to a three month time frame. Figures 7 and 8 investigate the symmetry of the response of the two volatility measures to changes in the underlying market.

FIGURE 7: WEEKLY CHANGE IN HISTORICAL VOL VS. WEEKLY CHANGE IN TOP40



As with the VIX, the perception of risk on the upside and downside for the SIVX is very similar, when using the historical volatility measure.

FIGURE 8: WEEKLY CHANGE IN SIVX VS. WEEKLY CHANGE IN TOP40

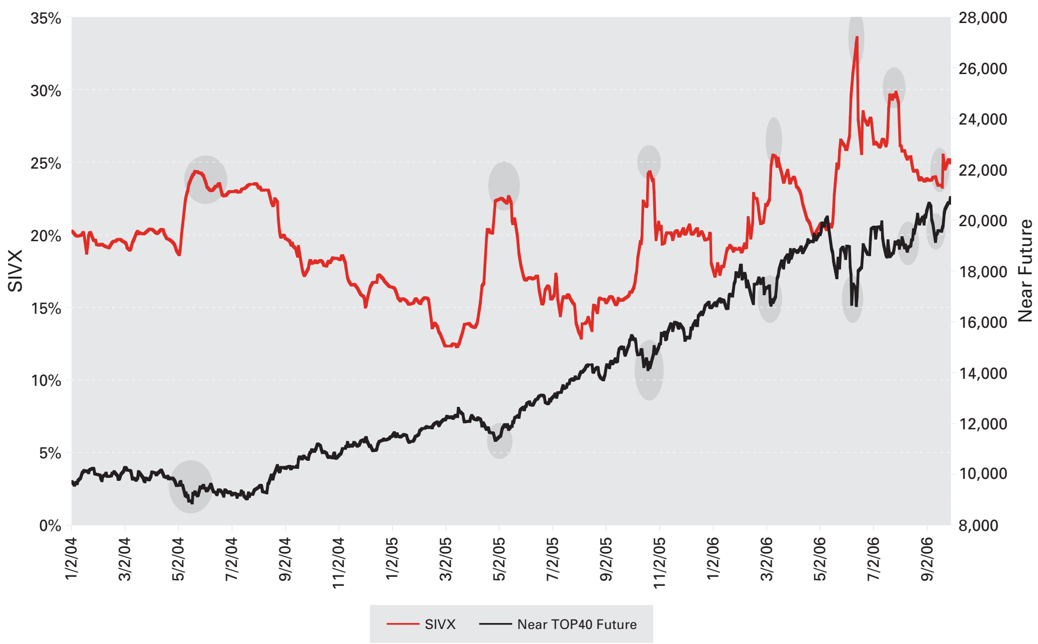
The implied volatility results are consistent with the more mature VIX but not as pronounced. The SIVX shows a non-symmetric response to changes in the market suggesting an expectation of higher risk in downward markets and lower levels of risk in upward markets.

6. SIVX APPLICATIONS

DIRECTIONAL INDICATOR/TIMING TOOL

Hill and Rattray (2004) investigated the idea that the VIX could be used as a successful timing indicator. They found in particular that peaks in the VIX tend to coincide with troughs in the market. We tested this phenomenon on the newly constructed SIVX and came up with the same result – spikes in the SIVX are accompanied by turning points in the TOP40 (signalling a buying opportunity). The TOP40 typically rallies after sharp upward moves in implied volatility, as illustrated in Figure 9.

FIGURE 9: SIVX VS. TOP40 NEAR FUTURE



Low levels of the SIVX do not, however, imply a weaker TOP40 market.

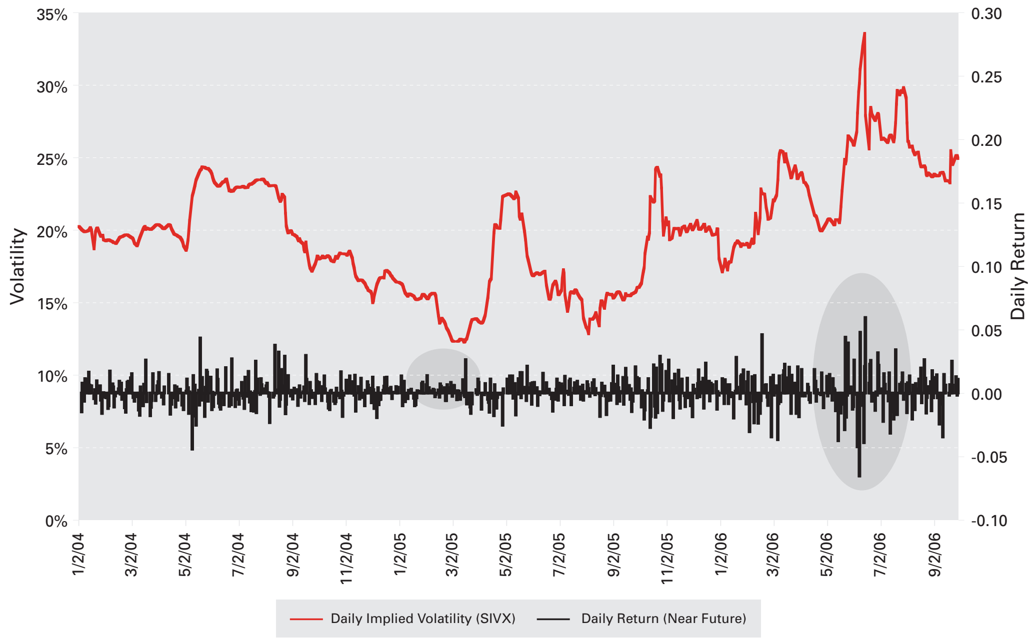
FEAR GUAGE

Montier (2004) claims that: "The VIX represents the consensus of option traders' forecasts for the S&P volatility over the next 30 days ... it is a reflection of Fear and Greed amongst a group of market professionals".

Hill and Rattray (2004) mention that the VIX is "constantly quoted as a market fear indicator in popular financial press – CNBC, Barrons, Wall Street Journal, etc".

The SIVX can be interpreted as a fear gauge too. This is seen graphically by plotting daily returns for the TOP40 near future against the SIVX

FIGURE 10: SIVX VS. NEAR TOP40 FUTURES RETURNS



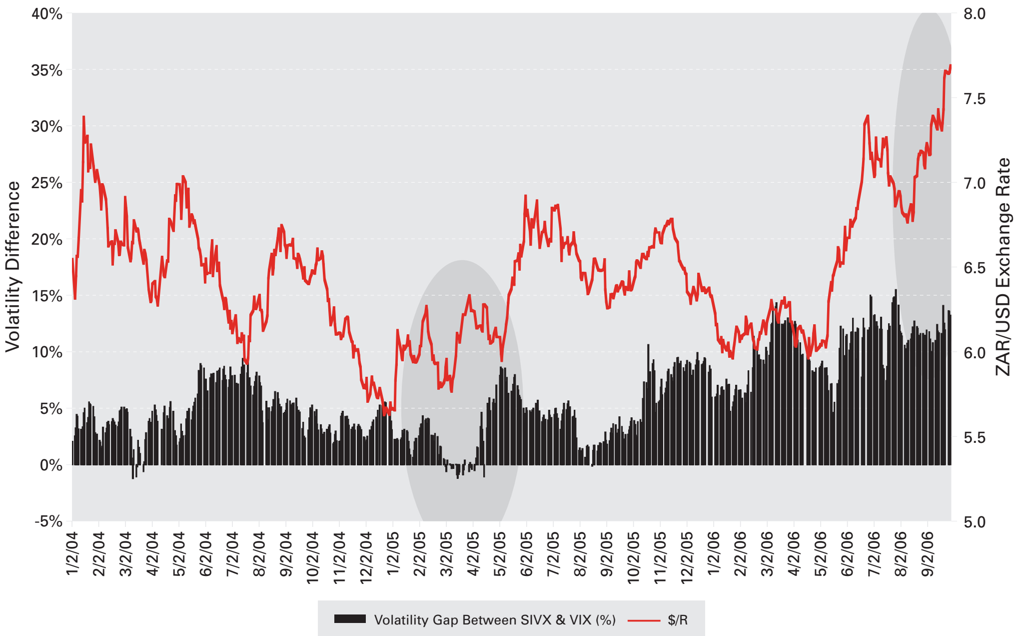
When daily returns are sufficiently volatile the SIVX tends to spike upward reflecting a higher level of implied volatility or expected risk. There is a greater sense of fear in the markets under these circumstances. A clustering of sufficiently low market returns is associated with low levels of volatility. This is reflected in equally low level of the SIVX. Under these circumstances the expectation/fear of risk is low.

COUNTRY/POLITICAL RISK MEASURE

The VIX is a "first- world" fear gauge and the SIVX is a similar gauge in an emerging market. Would the difference between these two forecast measures therefore not proxy the country risk or fear premium between an emerging market and a first world economy?

Figure 11 plots the difference between the SIVX and the VIX together with the USD/ZAR exchange rate.

FIGURE 11: DIFFERENCE BETWEEN SIVX AND VIX VS. USD/ZAR



To the extent that the volatility gap measures the expected increase in risk between the two countries we would want to see a high risk differential coincide with a weak rand and a low difference coincide with a strong Rand against the dollar. This is indeed the case.

Technically the VIX is a shorter term measure of risk than the SIVX. However, based on Figure 11 there still seems to be short term risk information embedded in the difference between these two measures.

SIVX AS AN UNDERLYING INSTRUMENT FOR A VOLATILITY FUTURE

A significant use of an implied volatility index would be its application to a volatility index futures contract. This has benefits for both asset managers and market makers in TOP40 index options. Such an application of an implied volatility index requires a lengthy discussion. A seperate report is dedicated to this topic.

CONCLUSION

In this report we discussed the construction and applications of the SIVX, an implied volatility index adapted to the South African market. The results were referenced against equivalent independent research performed on the worlds leading implied volatility index, the VIX. In this way the consistency of the SIVX results were tested and validated.

A summary of these results are as follows:

The SIVX can be used as:

- A fear gauge for an emerging market
- A timing tool for signalling entry points into the market
- A political/country risk measure
- An underlying “spot” instrument for a volatility future, the SIVXF

A solution is proposed that will enable the index to be published daily by SAFEX in a consistent manner by including inter-bank implied volatility levels in the index construction process.

REFERENCES

Black, K. "How the VIX Ate my Kurtosis", Illinois Institute of Technology, 2004

Bowler, B. "Volatility Products: Motivation and Mechanics", RISK Books, 2004, pp. 83 – 103

CBOE, "VIX CBOE Volatility Index", CBOE, 2003

Demeterfi K, E Derman, M Kamal, J Zou. "More Than You Ever Wanted To Know About Volatility Swaps", Goldman Sachs, March 1999

Derman, E et al. "Investing in Volatility", FOW, 1998

Fleming J, B Ostdiek, R Whaley, R. "Predicting Stock Market Volatility: A New Measure", The Journal of Futures Markets, Vol 15, No 3. 1995

Giot, P "Relationships Between Implied Volatility Indexes and Stock Index Returns", Institutional Investor, Spring 2005

Hill J and S Rattray. "Volatility as a Tradable Asset", Goldman Sachs & Co, March 2004

Montier, J. "Behavioural Finance", J Wiley & Sons, 2004, pp. 25 – 26

Neuberger, A. "The Log Contract and Other Power Contracts", The Handbook of Exotic Options, IRWIN, 1996, pp. 200 – 212

Perold, F and W Sharpe. "Dynamic Strategies for Asset Allocation", Financial Analysts Journal, Feb 1988

Shalen C and J Hiatt. "CBOE VIX Futures", RISK Books, 2004, pp.105 – 122

Skiadopoulos, G. "The Greek Implied Volatility Index: Construction and Properties", Draft, 2003

Taylor, S. "Asset Price Dynamics, Volatility and Prediction", Princeton, 2005, pp. 102 – 103

Whaley, R "The Investor Fear Guage", The Journal of Portfolio Management, Spring 2000

APPENDIX 1 – MEASURES OF VOLATILITY

We can quantify volatility using two measures:-

- *Realised Volatility* (weighted and un-weighted) – sometimes referred to as historical volatility
- *Implied Volatility* – sometimes decomposed into "local" volatility

The *realised volatility* of an index over some time period is a backward looking measure. Quantitatively speaking it is the annualised standard deviation of its daily returns over that time period given by the following equation,

$$\sigma = \sqrt{\frac{252}{n}\sum_{i=1}^{n}(r_i - \mu)^2}$$

where $\{r_i , i=1\ldots n\}$ are the returns and μ is the mean return for the past n days. The above equation is an un-weighted measure of realised volatility.

Weighted volatility measures are those that incorporate recent price change information into the measure of volatility. Models based on this methodology include EWMA (Exponentially Weighted Moving Average) and GARCH (General Autoregressive Conditional Heteroskedasticity).

EWMA volatility at time t is given by the following equation,

$$\sigma_t = \sqrt{\lambda\sigma_{t-1}^2 + (1-\lambda)(r_t - \mu)^2}$$

where $\lambda \varepsilon$ (0,1).

GARCH volatility is a more sophisticated measure that builds on the EWMA model and is given by the following equation,

$$\sigma_t = \sqrt{\alpha\sigma_{t-1}^2 + \beta(1-\lambda)(r_t - \mu)^2 + \gamma}$$

where, the condition $\alpha + \beta < 1$ is required for stationarity. Note the GARCH model reduces to the EWMA model when $\gamma = 0$.

Both these models aim to "remember" recent returns more, as opposed to those that are further back in time.

The *implied volatility* of an index is the volatility which equates the Black-Scholes value of an option to its market price. In essence, it is a forecast of the volatility of returns on the underlying asset concerned – over the life of the option. This volatility measure is numerically implied from an options price – no closed for solution exists for the implied volatility of an option.

For completeness, we briefly mention that implied volatility comprises more granular components called local volatilities. Derman (1998) discusses this in more detail – he focuses on the decomposition of the implied volatility term structure into local volatility measures in the same way as an interest rate can be decomposed into a strip of forward rates or "local" rates. A similar view is expressed by Shalen et al (2004): "The relationship between an implied volatility index and its future is analogous to the relationship between a spot rate of interest and a forward rate of interest". In this way, a volatility index future is a local volatility measure too.

APPENDIX 2 – PRICING THE SIVX

- T_1 = Actual days to Near Top40 Futures expiry
- $\sigma(0, T_1)$ = Implied ATM volatility of options on Near Top40 Futures
- T_2 = Actual days to Next-Near Top40 Futures expiry
- $\sigma(0, T_2)$ = Implied ATM volatility of options on Next Near Top40 Futures

Using CBOE (2003) calculate the SIVX by interpolating between the Near and Next-Near ATM implied volatilities to create a "ninety one" calendar day or N_T = 91 – 2 x INT(91/7) = 65 "trading day" implied volatility, ie.

$$SIVX = \sigma(0,T_1)\left[\frac{N_{T_2}-65}{N_{T_2}-N_{T_1}}\right] + \sigma(0,T_2)\left[\frac{65-N_{T_1}}{N_{T_2}-N_{T_1}}\right]$$

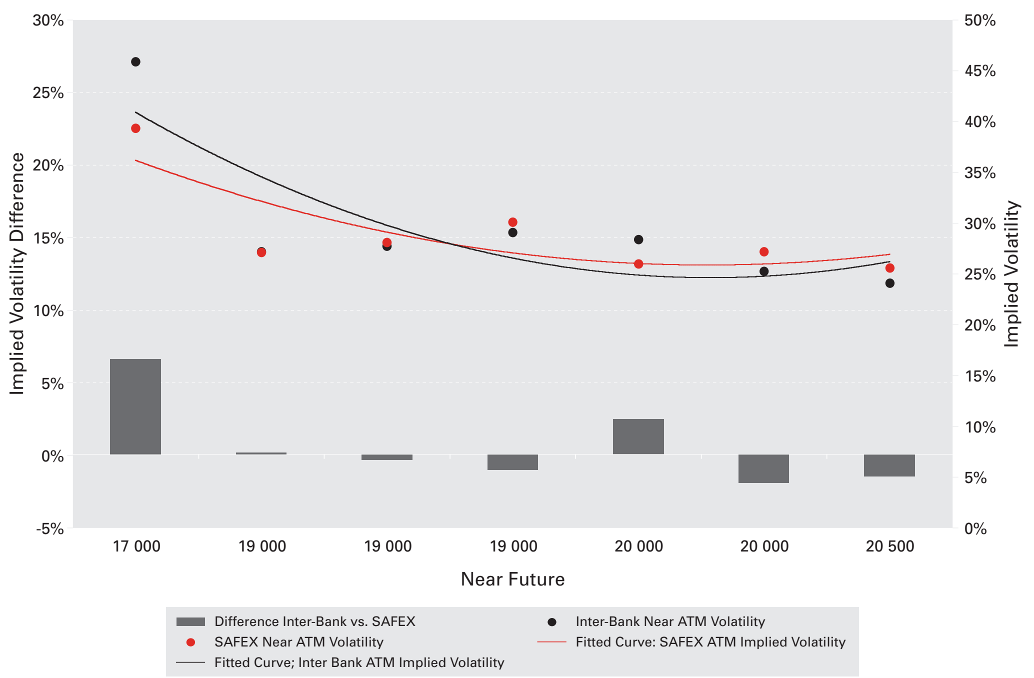
where N_{T_i} are the number of trading days till T_i.

APPENDIX 3 – COMPARISON BETWEEN SAFEX AND INTER-BANK VOLATILITY

This appendix looks at the similarity between implied volatilities obtained from:

- A small sample of SAFEX options, actively traded in the inter-bank market (as reported by a broker) versus,
- The implied volatilities reported by SAFEX for the exact options on the same day. These SAFEX implied volatilities are derived from the equivalent mark-to-market MTM option prices and MTN futures price for the day concerned.

FIGURE 16: SAFEX IMPLIED VOLATILITY VS. INTERBANK VOLATILITY



The left hand axis measures the difference between the MTM implied volatility and the "broker" volatility for the strike tested (the actual market). The histogram illustrates the "error". If the error is positive is suggests that the SAFEX MTM level underestimated the "actual" market. If it is negative then SAFEX overestimated the market implied volatility level for the strike concerned.

The right hand axis plots the actual implied volatility recorded by the "broker" or inter-bank market (black dots) and SAFEX MTM (red dots).

Fitted lines are plotted to illustrate the potential shape difference in the implied volatility term structures between SAFEX and the inter-bank market. Errors between the two are primarily driven by the MTM futures level used to "back out" the implied-volatility from the MTM option price.

PREVIOUS REPORTS

Improving on Correlation: Applications of Cointegration – Equity Pairs and Futures
Grant Shannon, July 2004

Futures Trading: A Basket Case – Cointegration, Futures Baskets, Spreads and Related Portfolios
Grant Shannon and Petrus Bosman, November 2004

Back to the Future: Dynamically Hedging Portfolios with SAFEX Share Futures
Grant Shannon and Petrus Bosman, June 2005

View Implementation: Testing the Barriers – Using Barrier Options in Equity Portfolios
Petrus Bosman and Grant Shannon, September 2005

To Swap Or Not To Swap – Equity Derivative Swaps and Asset Management
Grant Shannon and Petrus Bosman, November 2005

Equity Futures Not Currency Futures – Using Equity Futures to Manage Currency Risk in Internationally Diversified Portfolios
Grant Shannon and Petrus Bosman, March 2006

Common Sense In Common Trends – Using PCA to Build Long – Short Futures Portfolios
Grant Shannon and Manoshan Pillay, August 2006

CADIZ SECURITIES

DAN AHERN
Tel: 27 21 657 8500
e-mail: dan.ahern@cadiz.co.za

KEVIN MORLEY
Tel: 27 21 657 8550
e-mail: kevin.morley@cadiz.co.za

QUANTITATIVE RESEARCH
PROFESSOR DAVE BRADFIELD
Tel: 27 21 657 8351
e-mail: david.bradfield@cadiz.co.za

STOCKBROKING
DARREN SIMPSON
Tel: 27 11 853 8035
e-mail: darren.simpson@cadiz.co.za

EQUITY DERIVATIVES
Thabo Tembo
Tel: 27 21 657 8506
e-mail: thabo.tembo@cadiz.co.za

FIXED INCOME
Rick Kerwin
Tel: 27 21 657 8553
e-mail: rick.kerwin@cadiz.co.za

CADIZ CAPE TOWN OFFICE
Fernwood House, 1st Floor, The Oval
1 Oakdale Rd, Newlands 7700
P O Box 44547, Claremont 7735
Tel: +27 21 657 8300
Fax: +27 21 657 8303
info@cadiz.co.za

CADIZ JOHANNESBURG OFFICE
31 West Street, Lower Houghton, Johannesburg 2198
Postnet Suite 164, Private Bag X2600, Houghton 2041
Tel: +27 11 853 8000
Fax: +27 11 483 0869
info@cadiz.co.za