PRICING METHODOLOGY FOR INFLATION LINKED BONDS AND INFLATION LINKED BUY/SELL-BACKS

This document provides the specification for pricing inflation linked bonds listed on the Bond Exchange of South Africa (BESA) as well as pricing Inflation Linked Buy/Sell-Backs.

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INTRODUCTION

This document outlines the manner in which BESA will calculate the all-in price of the inflation linked bonds as well as Inflation Linked Buy/Sell-Backs.

We know that inflation is a key driver of bond performance and a fundamental component of yield to maturity for vanilla fixed coupon bonds. It is important to note that yield to maturity is a nominal return which can be de-composed into an inflation and real return component. Therefore when one trades a vanilla fixed coupon bond, you are implicitly making an assumption as to what you believe future inflation is going to be. In order to measure true performance of an investment, the “real return” is calculated. Many issuers recognised this and devised a bond (the inflation linked bond) based on “Real Yield” rather than “Nominal Yield” with the promise to protect purchasing power.

Inflation linked bonds were first issued by the South African Government on 20 March 2000. These bonds have been issued such that both their coupons and principal are linked to the South African Consumer Price Index (CPI) as distributed by Statistics South Africa (“Stats SA”). Please note that it is the headline CPI that is used (i.e. the Consumer Price Index for the historical metropolitan areas –all items) and not CPIX (Consumer Price Index excluding interest rates on mortgage bonds). The historical CPI information can be found at www.statssa.gov.za
In summary, the method is as follows:

1. Determine the settlement date for the trade;
2. Determine the Reference CPI (see definition in section 3) for the issue date;
3. Determine the Reference CPI (see definition in section 3) for the settlement date
4. Use the Reference CPI for the settlement date, as well as the Reference CPI for the issue date to calculate the Index Ratio;
5. Use the standard BESA bond pricing formula with the “real yield” instead of the nominal yield to obtain a value;
6. Multiply the value obtained in sub item 5 above by the Index Ratio to obtain an all-in price of the inflation indexed bond; and
7. Round the result obtained in sub item 6.

We describe each step in more detail below.
3 Methodology to Calculate the All-in Price of the Inflation Linked Bond

In order for an issuer to compensate investors for inflation over time, the inflation figure has to be determined on the date of issue and measured over the required period. (Please note that the date of issue is the settlement date of when the bond is issued and not the trade date.)

Unfortunately, the CPI figure applies for a period of a month and not a specific day in a month. As a result, we assume that when the CPI figure comes out for any particular month, the CPI figure will only apply for the first calendar day of each respective month. In order to calculate the CPI figure applicable for any other day in a month; we simply linearly interpolate between the CPI figures of each month. For example if the CPI figure for January is 100 and the CPI for February is 102, the middle of January would be approximately 101.

Please note that the CPI figure is generated retrospectively and as a result there is a time lag as to when this figure becomes publicly available for the month in question; i.e. the inflation figure for January may only be available in March. The inflation figure applicable to any particular settlement date will be based on the interpolated CPI figures looking back 4 months and 3 months respectively. Therefore programmers must note that if a user was to enter into a 3 month forward dated inflation linked bond trade, the applicable CPI figures would probably not yet be available.

3.1 Determine the settlement date

The settlement will usually be based on the “t+3” convention, i.e. trade date plus three business days. This is in accordance with the standard South African bond market convention, but is not a requirement.

3.2 Determine the Reference CPI for the Inflation Linked Bond’s Issue Date

The methodology for determining the Reference CPI for the inflation linked bond on issue date is exactly the same as that for any other date. Essentially we need to know what the inflation figure is for the settlement date of issue. This is done as follows:

“Reference CPI” or “REFCPI_{Issue \text{ Date}}” means, in relation to the settlement date on which the issue took place:
if the issue date is the first day of a calendar month, \( \text{REFCPI}_{\text{Issue \_ Date}} \) is the Consumer Price Index for the fourth calendar month preceding the calendar month in which issue date occurs;

- if the issue date occurs on any day other than the first day of any calendar month, then \( \text{REFCPI}_{\text{Issue \_ Date}} \) shall be determined in accordance with the following formula:

\[
\text{REFCPI}_{\text{Issue \_ Date}} = CPI_j + \left[ \frac{(t-1)}{D} \times (CPI_{j+1} - CPI_j) \right]
\]

Where:

- \( CPI_j \) is the CPI figure for the first day of the calendar month 4 months preceding the issue date;

- \( CPI_{j+1} \) is the CPI figure for the first day of the calendar month 3 months preceding the issue date;

- \( t \) is the calendar day corresponding to issue date; and

- \( D \) is the number of days in the calendar month in which issue date occurs.

If a required CPI value is not available due to a delay in publication of the CPI, or where there are adjustments to the CPI, then please refer to the respective issuer listing document for more information.

### 3.3 Determine the Reference CPI for the settlement date

This process is identical to that of determining the Reference CPI on Issue date.

Again, this is done as follows:

“Reference CPI” or “\( \text{REFCPI}_{\text{Sett \_ Date}} \)” means, in relation to the settlement date on which the trade took place:

- if the settlement date is the first day of a calendar month, \( \text{REFCPI}_{\text{Sett \_ Date}} \) is the Consumer Price Index for the fourth calendar month preceding the calendar month in which settlement date occurs;

- if the settlement date occurs on any day other than the first day of any calendar month, then \( \text{REFCPI}_{\text{Sett \_ Date}} \) shall be determined in accordance with the following formula:

\[
\text{REFCPI}_{\text{Sett \_ Date}} = CPI_j + \left[ \frac{(t-1)}{D} \times (CPI_{j+1} - CPI_j) \right]
\]

Where:
• $CPI_j$ is the CPI figure for the first day of the calendar month 4 months preceding the settlement date;
• $CPI_{j+1}$ is the CPI figure for the first day of the calendar month 3 months preceding the settlement date;
• $t$ is the calendar day corresponding to settlement date; and
• $D$ is the number of days in the calendar month in which settlement date occurs.

3.4 Calculate the Index Ratio

The Index Ratio is calculated by dividing the Reference CPI for the settlement date by the Reference CPI for the issue date.

\[
Index\text{ratio} = \frac{REFCPI_{Set\_Date}}{REFCPI_{Issue\_Date}}
\]

This calculation is subject to the condition that, on the redemption date of the bonds, the Index Ratio will be no less than one.

3.5 BESA Bond Pricing Formula

The BESA Bond Pricing Formula is used to obtain a value. This formula is defined in Equation 6 of the document “Bond Pricing Formula Specifications”, dated 24 August 2005. A copy of this document can be downloaded from BESA’s website - http://www.bondexchange.co.za.

The way in which the formula is used here is slightly different from the way in which the formulae are normally used. The differences are:

• a “real yield” is used in place of the “yield-to-maturity”; and
• the coupon that is used is the coupon of the inflation indexed bond.

Please note that this inflation linked specification assumes that the attributes of the inflation linked bonds are similar to those of standard fixed-coupon bonds. In particular:

• Coupons will be paid bi-annually and one of the coupon payment dates will coincide with the anniversary of the bond’s maturity date.
• Book closing dates and coupon dates will be published for each inflation indexed bond and the bond becomes ex-coupon on the book closing date.
• The entire capital of the bond will be redeemed at maturity date (there is an exception for certain multiple maturity bonds.)

3.6 Calculate the All-in Price of the Inflation Linked Bond

The rounded result obtained from the BESA bond pricing formula in (3.5) is multiplied with the unrounded Index Ratio obtained in (3.4), in order to obtain an unrounded result.
3.7 Rounding Conventions

- All intermediate calculations are to IEEE double precision (15 significant digits).
- The all-in price of the inflation indexed bond is obtained by multiplying the rounded all-in price from the BESA bond formula by the unrounded Index Ratio, and the result is rounded to 5 decimal places.
- The accrued interest is calculated by multiplying the rounded accrued interest obtained from the BESA bond pricing formula by the unrounded Index Ratio. The result is rounded to 5 decimal places.
- The rounded clean price is obtained by subtracting the rounded accrued interest from the rounded all-in price.

3.8 Delta

Delta for the inflation indexed Bond is simply the product of delta as per the “Bond Pricing Formula Specifications” with the index ratio.

\[ \Delta_{\text{CPI}} = \Delta_{\text{vanilla}} \times \text{Indexratio} \]

Where:

- \( \Delta_{\text{vanilla}} \) = Delta calculated by the BESA Pricing Formula (referenced above) using the real yield as per section 3.5 above.

3.9 Duration

Modified Duration for a vanilla bond is simply defined as:

\[ D_{\text{mod}} = -\frac{100 \times \text{Delta}}{\text{AIP}} \]

Modified duration for a CPI bond is therefore

\[ D_{\text{mod CPI}} = -\frac{100 \times \Delta_{\text{CPI}}}{\text{AIP}_{\text{unrounded CPI}}} = -\frac{100 \times \Delta_{\text{vanilla}} \times \text{Indexratio}}{\text{AIP}_{\text{vanilla}} \times \text{Indexratio}} = -\frac{100 \times \Delta_{\text{vanilla}}}{\text{AIP}_{\text{unrounded vanilla}}} \]

i.e. \( D_{\text{mod CPI}} = D_{\text{mod vanilla}} \)

Where:

- \( \text{AIP}_{\text{unrounded CPI}} \) = This is the unrounded CPI All-in Price calculated in sub item 3.6.
\[ \text{DELTA}_{\text{vanilla}} = \text{DELTA calculated by the BESA Pricing Formula (referenced above) using the real yield as per section 3.5 above.} \]

Macaulay’s duration is derived as per the “Bond Pricing Formula Specifications” referenced above i.e.
\[ D_{\text{CPI}} = \frac{D_{\text{mod CPI}}}{F} \]

where:
\[ D_{\text{CPI}} = \text{Macaulay Duration CPI figure. Since } D_{\text{mod CPI}} = D_{\text{mod vanilla}}, \text{ therefore } D_{\text{CPI}} = D_{\text{vanilla}} \]
\[ F = \left( 1 + \frac{y}{200} \right)^{-1} \]
i.e. Semi-annual discount factor corresponding to the real yield \( Y \)

### 3.10 Convexity

Convexity is defined as:
\[ \text{Convexity} = \frac{10000}{AIP_{\text{rounded}}} \times \frac{\partial^2 AIP}{\partial Y^2} \]

Convexity for a CPI bond is therefore:
\[ \text{Convexity}_{\text{CPI}} = \frac{10000}{AIP_{\text{vanilla}} \times \text{Indexratio}} \times \frac{\partial^2 AIP_{\text{vanilla}}}{\partial Y^2} \times \text{Indexratio} = \frac{10000}{AIP_{\text{vanilla}}} \times \frac{\partial^2 AIP_{\text{vanilla}}}{\partial Y^2} \]

Where:
\[ \frac{\partial^2 AIP_{\text{vanilla}}}{\partial Y^2} = \text{the second differential of vanilla AIP with respect to yield as per the “Bond Pricing Formula Specifications” referenced, using the real yield as per section 3.5 above.} \]
What is the Price of the R189 for settlement 10 October 2005 with a real yield of 2.7%?

Using the publically available table of CPI index values:

<table>
<thead>
<tr>
<th>Month</th>
<th>CPI value</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 1999</td>
<td>95.5</td>
</tr>
<tr>
<td>December 1999</td>
<td>95.8</td>
</tr>
<tr>
<td>May 2005</td>
<td>127.6</td>
</tr>
<tr>
<td>June 2005</td>
<td>127.4</td>
</tr>
<tr>
<td>July 2005</td>
<td>128.5</td>
</tr>
</tbody>
</table>

The R189 details are as follows:

<table>
<thead>
<tr>
<th>R189 Details</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Coupon Rate</td>
<td>6.25</td>
</tr>
<tr>
<td>Maturity</td>
<td>31 March 2013</td>
</tr>
<tr>
<td>Issue date</td>
<td>20 March 2000</td>
</tr>
<tr>
<td>Interest Payable1</td>
<td>31-Mar</td>
</tr>
<tr>
<td>Interest Payable2</td>
<td>30-Sep</td>
</tr>
<tr>
<td>Books Closed1</td>
<td>21-Mar</td>
</tr>
<tr>
<td>Books Closed2</td>
<td>20-Sep</td>
</tr>
</tbody>
</table>

To convert this real yield to a price we have to first determine the Reference CPI value for this bond on settlement date as well as when the bond was issued. This is given by:

**Reference CPI on issue date:**

\[\text{CPI}_{\text{Nov-1999}} + \frac{19}{31} (\text{CPI}_{\text{Dec-1999}} - \text{CPI}_{\text{Nov-1999}}) = 95.5 + \frac{19}{31} (95.8 - 95.5) = 95.6838709677419\ldots\]

**Reference CPI on Settlement date:**

\[\text{CPI}_{\text{June-2005}} + \frac{9}{31} (\text{CPI}_{\text{July-2005}} - \text{CPI}_{\text{June-2005}}) = 127.4 + \frac{9}{31} (128.5 - 127.4) = 127.71935483871\ldots\]

**The Index Ratio is given by:**

\[
\text{Index Ratio} = \frac{127.71935483871}{95.6838709677419} = 1.33480547501854\ldots
\]

The BESA bond pricing formula yields R124.04813 for the R189 trading at a yield of 2.7% for settlement on 10 October 2005. The inflation indexed bond price is obtained by multiplying this price by the Index Ratio. Therefore the inflation indexed All-in bond price is **R165.58012**.

The Clean Price is R165.35156 and the Accrued Interest is: R0.22856

Delta is: -10.1930

Modified Duration is: 6.156

Duration is: 6.239

Convexity is: 45.347
5 Inflation Linked Buy/Sell Backs

This section explains how the Bond Exchange Trade Capture system calculates the prices for Inflation Linked Buy/Sell-Backs listed on the Exchange. For a more detailed overview of Pricing Repo's / Buy/Sell Back's in South Africa, please refer to the “Pricing Buy/Sell Backs” Document distributed by BESA.

Pricing Inflation Linked Buy/Sell-backs is not as simple as pricing vanilla Buy/Sell-backs. It is further complicated in South Africa because of the historical methodology we use for pricing Buy/Sell-backs whereby we iterate back a yield to calculate the consideration for the second leg of the Buy/Sell-backs.

5.1 Pricing Inflation Linked Buy/Sell-Backs

The calculation of the first leg of the Inflation Linked Buy/Sell-back is simply the All-in price of the Inflation linked bond as calculated in section 3.6 above \( AIP_{\text{CPI}} \). Calculation of the second leg of the Inflation Linked Buy/Sell-back is divided into four steps namely:

1. Adding the Repo interest to the All-in price calculated on the first settlement leg.
2. Adjusting for the impact of any future coupons.
3. Iterate back the closest yield corresponding to the calculated second leg All-in price.
4. Use this yield to recalculate the second leg of the Inflation Linked Buy/Sell-back.

5.1.1 Steps 1 & 2

\[
AIP_{\text{CPI}_{2}} = AIP_{\text{CPI}_{1}} \times \left(1 + \frac{r}{100} \times \frac{d_{2} - d_{1}}{365}\right) \times \frac{CPN}{2} \times \sum_{i} \left[EV_{cd_{i},d_{2}} \times \text{Indexratio}_{cd_{i}}\right]
\]

Where:

\[
EV_{cd_{1},d_{2}} = \frac{1}{1 + \frac{r}{100} \times \frac{cd_{i} - d_{2}}{365}} \quad \text{if } d_{2} < cd_{i} \text{ and } d_{2} \text{ is in the ex period.}
\]

\[
EV_{cd_{2},d_{2}} = 1 + \frac{r}{100} \times \frac{d_{2} - cd_{i}}{365} \quad \text{if } d_{2} \geq cd_{i}
\]

\( d_{1} = \text{First settlement date} \)

\( d_{2} = \text{Second Settlement date} \)
Note that $d_2 > d_1$ as per market convention.

\[ r = \text{Inflation Linked Buy/Sell-back rate (Repo rate) expressed as a simple rate. Please note that the interest rate of say 8% would be represented as 8.} \]

\[ cd_i = \text{This is the set of coupon dates which affect the Inflation Linked Buy/Sell-back. In other words the person who hands over the bonds as collateral in return for putting cash on deposit at the “Repo” rate is still entitled to receive the coupons from the collateral (our Inflation Linked bonds); we therefore account for these coupon payments in the second leg of the Buy/Sell-back. If there are no coupon consequences then } cd_i \text{ will be zero and we would ignore the second term in the equation. } i=1 \text{ refers to the next coupon date and } i=2 \text{ would refer to the following coupon date etc;} \]

\[ EV = \text{Equivalent value. If the second leg of the Inflation Linked Buy/Sell-back falls in the bond’s ex period, then we discount the coupon back to } d_2 \text{ and subtract it from the proceeds to be paid to the counterparty as they will receive the next coupon. If on the other hand, the second leg of the Inflation Linked Buy/Sell-back follows a coupon date and the counterparty receives the coupon, we subtract not only the value of the coupon but the interest we could have earned on the coupon using the “Repo” rate for the interest calculation. If the Inflation Linked Buy/Sell-back spans many coupon dates, we would subtract each respective coupon and the interest we could earn on each respective coupon up to the second leg of the Inflation Linked Buy/Sell-back.} \]

\[ \text{Indexratio}_{cd_i} = \text{This is the Index ratio applicable to the coupon payment date. If a coupon is paid, we multiply the coupon by the index ratio applicable to the coupon date and incorporate this in our final calculation i.e. we adjust the coupon for inflation (multiply by the index ratio) as this is the cash flow that would be received by the bondholder.} \]

\[ CPN = \text{Annual Coupon.} \]

5.1.2 Step 3

To iterate back the yield corresponding to $AIP_{CPI}$, we determine the Reference CPI on date 2 (second leg of the Inflation Linked Buy/Sell-back) as well as index ratio for the second leg. The iteration then tests for a real yield equal to the $AIP_{CPI}$, given the index ratio. For more information on the process of iteration, please refer to the “Bond Pricing Specification.”

5.1.3 Step 4

Since we have to iterate the real yield of the AIP on the second leg \settlement date taking into account the index ratio on this date (second settlement date), this results in a limitation in that if the Inflation Linked Buy/Sell-back is over 4 months, we will not have the data for the index ratio, simply because the statistic will not be available yet (inflation figures actually lag by approximately 4-6 weeks.) We use this yield iteration to calculate the final value of the All-in price of the second leg.
What are the prices of a R189 Inflation Linked Buy/Sell-back for settlement 15 March 2005 to 04 April 2005 with a “Repo” rate of 6.5% and the spot level of the R189 (for value 15 March 2005) is a real yield of 2.7%? Using the publically available table of CPI index values:

<table>
<thead>
<tr>
<th>Month</th>
<th>CPI value</th>
</tr>
</thead>
<tbody>
<tr>
<td>November 1999</td>
<td>95.5</td>
</tr>
<tr>
<td>December 1999</td>
<td>95.8</td>
</tr>
<tr>
<td>November 2004</td>
<td>125.3</td>
</tr>
<tr>
<td>December 2004</td>
<td>125</td>
</tr>
<tr>
<td>January 2005</td>
<td>125.4</td>
</tr>
</tbody>
</table>

The R189 details are as follows:

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<tr>
<td>Issue date</td>
</tr>
<tr>
<td>Interest Payable1</td>
</tr>
<tr>
<td>Interest Payable2</td>
</tr>
<tr>
<td>Books Closed1</td>
</tr>
<tr>
<td>Books Closed2</td>
</tr>
</tbody>
</table>

We have to first determine the Reference CPI value for this bond on 15 March settlement date as well as when the bond was issued. This is given by:

Reference CPI on issue date:

\[
CPI_{Nov1999} + 19/31 \left( CPI_{Dec1999} - CPI_{Nov1999} \right) = 95.5 + 19/31 (95.8 - 95.5) = 95.6838709677419...
\]

Reference CPI on Settlement date 15 March 2005:

\[
CPI_{Nov2004} + 14/31 \left( CPI_{Dec2004} - CPI_{Nov2004} \right) = 125.3 + 14/31 (125 - 125.3) = 125.164516129032...
\]

The Index Ratio is given by:

Index Ratio = 125.1645.. / 95.6838709677419= 1.30810464...

We also need to know the index ratio of the coupon date, i.e. when the coupon is paid. Reference CPI on coupon date:

\[ CPI_{Nov\,2004} + \frac{30}{31} \left( CPI_{Dec\,2004} - CPI_{Nov\,2004} \right) = 125.3 + \frac{30}{31} \left( 125 - 125.3 \right) \]
\[ = 125.009677419355... \]

The Index Ratio on the coupon date is given by:

Index Ratio = \[ \frac{125.009677...}{95.6838709677419} = 1.30648641359315... \]

Therefore, we can now calculate the All-in – price for the second leg of the Inflation Linked Buy/Sell-back:

We now need to calculate the Reference CPI on the 04 April 2005 and iterate a real yield using the index ratio for this date to find the closest All-in price to the already calculated above.

Reference CPI on 04 April:

\[ CPI_{Dec\,2004} + \frac{3}{30} \left( CPI_{Jan\,2005} - CPI_{Dec\,2004} \right) = 125 + \frac{3}{30} \left( 125.4 - 125 \right) = 125.04 \]

The Index Ratio on the coupon date is given by:

Index Ratio = \[ \frac{125.04}{95.6838709677419} = 1.30680331737... \]

The iteration then produces the closest real yield of:

\[ \begin{align*}
AIP_{CPI_{2}} &= R167.91173 \times \left(1 + \frac{0.065 \times 20}{365}\right) - 3.125 \times \left(1 + \frac{0.065 \times 4}{365}\right) \times 1.30648641359315 = R164.42409346105.. \end{align*} \]

We now need to calculate the Reference CPI on the 04 April 2005 and iterate a real yield using the index ratio for this date to find the closest All-in price to the already calculated above.

Reference CPI on 04 April:

\[ CPI_{Dec\,2004} + \frac{3}{30} \left( CPI_{Jun\,2005} - CPI_{Dec\,2004} \right) = 125 + \frac{3}{30} (125.4 - 125) = 125.04 \]

The Index Ratio on the coupon date is given by:

Index Ratio = \[ \frac{125.04}{95.6838709677419} = 1.30680331737... \]

The iteration then produces the closest real yield of:

\[ \begin{array}{|c|c|}
\hline
2.65165 & 164.42398 \\
\hline
2.65164 & 164.42410 \\
\hline
2.65163 & 164.42420 \\
\hline
\end{array} \]

Therefore the \[ AIP_{CPI_{2}} \] of R164.42410 is used to calculate the consideration of the second leg of the Inflation Linked Buy/Sell-back as it is the closest iteration.
**Example 2**

What are the prices of a R189 Inflation Linked Buy/Sell-back for settlement 15 March 2005 to 29 March 2005 with a "Repo" rate of 6.5% and the spot level of the R189 (for value 15 March 2005) is a real yield of 2.7%? Using the same information as example 1, the calculations of the Reference CPI on Settlement date 15 March 2005 and the Index ratio for this date and the Reference CPI on coupon date are identical.

Therefore, we can now calculate the All-in price for the second leg of the Linked Buy/Sell-back:

\[
AIP_{CPI_2} = R167.91173 \times \left(1 + \frac{0.065 \times 14}{365}\right)^{-3.125 \times \left(1 + \frac{1}{1 + \frac{0.065 \times 2}{365}}\right)^{1.30648641 \times 35931} = 164.249042 \ 821727...
\]

We now need to calculate the Refcpi on the 29 March 2005 and iterate a real yield using the index ratio for this date to find the closest All-in price to the \( AIP_{CPI_2} \) already calculated above.

Reference CPI on 29 March 2005:

\[
CPI_{Nov2004} + 28/31 \left( CPI_{Dec2004} - CPI_{Nov2004} \right) = 125.3 + 28/31 (125 - 125.3) = 125.029032258...
\]

The Index Ratio on the 29 March is:

Index Ratio = 125.029032258 / 95.6838709677419 = 1.306688692603...

The iteration then produces the closest real yield of:

| 2.65997 | 164.24914 |
| 2.65998 | 164.24903 |
| 2.65998 | 164.24893 |

Therefore the \( AIP_{CPI_2} \) of R164.24903 is used to calculate the consideration of the second leg of the Inflation Linked Buy/Sell-back as it is the closest iteration.
Example 3

What are the prices of a R189 Inflation Linked Buy/Sell-back for settlement 21 March 2005 to 10 April 2005 with a “Repo” rate of 6.5% and the spot level of the R189 (for value 21 March 2005) is a real yield of 2.7%? Using the data from the table and the calculations from the previous examples:

Reference CPI on 21 March 2005:

\[
CPI_{Nov2004} + \frac{20}{31} (CPI_{Dec2004} - CPI_{Nov2004}) = 125.3 + \frac{20}{31} (125 - 125.3) \\
= 125.10645161\ldots
\]

The Index Ratio on the 21 March is:

\[
\text{Index Ratio} = \frac{125.106451613}{95.6838709677419} = 1.307497808644\ldots
\]

The BESA bond pricing formula yields R 125.29667 for the R189 trading at a yield of 2.7% for settlement on 21 March 2005. Therefore the inflation indexed bond price is R**163.82512** for the 21 March 2005.

\[
AIP_{CPI} = R163.82512 \times \left(1 + \frac{0.065}{365} \times 20\right) = R164.40860672 \ldots
\]

Since there are no coupons involved, as the seller receives the coupon, the formula is considerably simpler. We now need to calculate the Reference CPI on the 10 April 2005 and iterate a real yield using the index ratio for this date to find the closest All-in price to the **$AIP_{CPI}$** already calculated above.

Reference CPI on 10 April 2005:

\[
CPI_{Dec2004} + \frac{9}{30} (CPI_{Jan2005} - CPI_{Dec2004}) = 125 + \frac{9}{30} (125.4 - 125) = 125.12
\]

The Index Ratio on the 10 April 2005 is:

\[
\text{Index Ratio} = \frac{125.12}{95.6838709677419} = 1.30763940395\ldots
\]

The iteration then produces the closest real yield of:

<table>
<thead>
<tr>
<th>2.66956</th>
<th>164.40872</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.66957</td>
<td>164.40861</td>
</tr>
<tr>
<td>2.66958</td>
<td>164.40851</td>
</tr>
</tbody>
</table>
Therefore the $AIP_{CPI}$ of R164.40861 is used to calculate the consideration of the second leg of the Inflation Linked Buy/Sell-back as it is the closest iteration.

**Example 4**

What are the prices of a R189 Inflation Linked Buy/Sell-back for settlement 21 March 2005 to 29 March 2005 with a “Repo” rate of 6.5% and the spot level of the R189 (for value 21 March 2005) is a real yield of 2.7%? Using the data from the table and the calculations from the previous examples (including the first All-in Price):

\[
AIP_{CPI} = R163 \times \left( 1 + \frac{0.065 \times 8}{365} \right) = R164 \quad 0.0585146.. 
\]

Again, the calculation of the second leg All-in-price is considerably simplified as no coupons are involved in the transaction.

Reference CPI on 29 March 2005:

\[
CPI_{Nov2004} + \frac{28}{31} \left( CPI_{Dec2004} - CPI_{Nov2004} \right) = 125.3 + \frac{28}{31} (125 - 125.3) \\
= 125.029032258...
\]

The Index Ratio on the 29 March is:

Index Ratio = 125.029032258 / 95.6838709677419 = 1.306688692603...

The iteration then produces the closest real yield of:

| 2.6774   | 164.05859 |
| 2.6775   | 164.05848 |
| 2.6776   | 164.05837 |

Therefore the $AIP_{CPI}$ of R164.05848 is used to calculate the consideration of the second leg of the Inflation Linked Buy/Sell-back as it is the closest iteration.